Extended parallel backprojection for standard three-dimensional and phase-correlated four-dimensional axial and spiral cone-beam CT with arbitrary pitch, arbitrary cone-angle, and 100% dose usage

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We have developed a new approximate Feldkamp-type algorithm that we call the extended parallel backprojection (EPBP). Its main features are a phase-weighted backprojection and a voxel-by-voxel 180° normalization. The first feature ensures three-dimensional (3-D) and 4-D capabilities with one and the same algorithm; the second ensures 100% detector usage (each ray is accounted for). The algorithm was evaluated using simulated data of a thorax phantom and a cardiac motion phantom for scanners with up to 256 slices. Axial (circle and sequence) and spiral scan trajectories were investigated. The standard reconstructions (EPBPStd) are of high quality, even for as many as 256 slices. The cardiac reconstructions (EPBPCI) are of high quality as well and show no significant deterioration of objects even far off the center of rotation. Since EPBPCI uses the cardio interpolation (CI) phase weighting the temporal resolution is equivalent to that of the well-established single-slice and multislice cardiac approaches 180°CI, 180°MCI, and ASSRCI, respectively, and lies in the order of 50 to 100 ms for rotation times between 0.4 and 0.5 s. EPBP appears to fulfill all required demands. Especially the phase-correlated EPBP reconstruction of cardiac multiple circle scan data is of high interest, e.g., for dynamic perfusion studies of the heart. © 2004 American Association of Physicists in Medicine. [DOI: 10.1118/1.1755569]

Key words: Cone-beam CT (CBCT), cardiac imaging, 4-D reconstruction, image quality

I. INTRODUCTION

The ongoing development of medical cone-beam CT (CBCT) scanners requires providing cone-beam reconstruction algorithms adequate for medical purposes. These must be capable of handling arbitrary pitch standard and phase-correlated reconstruction for circular, sequential and spiral trajectories. (Flexible pitch selection in the range from about 0.3 to about 1.5 is used for dynamic studies to have full control over the table speed, and it is used for cardiac CT to adopt the amount of data overlap to the patient’s heart rate. One further adjusts the pitch to compensate for tube power limits: low pitch values allow us to accumulate dose but increase the total scan time. Finding the best compromise requires flexibility in pitch selection; two or three distinct pitch values are certainly not enough.) Due to the increasing dose awareness, it is further of importance to ensure 100% dose usage, which means that each measured ray contributes to the reconstructed images.

As far as we know, three types of image reconstruction algorithms can be found in today’s 16-slice medical CT scanners: z-interpolation, single-slice rebinning, and Feldkamp-type image reconstruction. To reconstruct stationary objects (standard reconstruction) manufacturers use Feldkamp-based algorithms1–6 or they use the so-called single-slice rebinning algorithms such as the advanced single-slice rebinning algorithm (ASSR).7–12 The reconstruction of periodically-moving objects such as the beating heart is done with phase-correlated algorithms. There, two-dimensional z-interpolation algorithms dedicated for four-slice CT are adopted and used to reconstruct cardiac data for scanners with more than four slices.

However, there are several restrictions to these approaches that may inhibit their use in scanners with significantly more than 16 slices. It is known that the Feldkamp-type reconstruction of spiral data is of acceptable quality only if the direction of convolution corresponds to the direction of the spiral tangent.13 Whereas this fact can be accounted for, a severe drawback is the use of a detector window similar to the Tam window14 that results in pitch-dependent dose efficiency values that are always significantly below 100%. The ASSR algorithm yields good image quality for scanners with up to 64 slices2 and has been generalized for arbitrary pitch spiral CT with 100% dose usage.15 Its generalization to cardiac scanning ASSRCI (cardio interpolation) yields good images only for as little as 16 to 32 slices.15

Besides these restrictions, the development of wide cone angle medical CT scanners with large area detectors (64 slices or more) will likely draw attention toward the circular scan trajectory. Especially cardiac CT is likely to push the development of reconstruction algorithms for circle scans. Assume a scanner with $M = 256$ slices and a slice thickness of $S = 0.75$ mm. The total collimated width of $MS = 192$ mm is sufficient to cover the heart completely and a circle scan will be the trajectory of choice. This scan mode would allow us to increase temporal resolution simply by increasing scan overlap, which directly corresponds to the
scan time or the number of rotations used for a circle scan.\textsuperscript{16} Even more important, the circle scan is the trajectory required for dynamic studies such as perfusion measurements.\textsuperscript{17,18}

To accommodate these demands, we developed and evaluated the extended parallel backprojection (EPBP), which is a new class of standard and phase-correlated image reconstruction algorithms that handle circular scans, sequential scans and spiral scans with arbitrary pitch and 100\% dose usage.\textsuperscript{19,20} EPBP is a Feldkamp-type algorithm that performs azimuthal, longitudinal, and radial rebinning to parallel geometry before an extended 3-D cone-beam backprojection with voxel-dependent weights generates the final volume.

Rebinning to parallel geometry using a slanted detector (longitudinal rebinning) has been discussed in the literature before. An example are the PI and the PI-SLANT algorithms.\textsuperscript{21,22} However, these do not achieve a full detector utilization nor do these approaches allow for phase-correlated reconstruction. Detector utilization itself has become an issue recently. For example, Ref. 23 proposes a new weighting scheme to improve the detector usage and to reduce image noise; 100\% dose usage is not achieved, however. Also recently introduced was the extended cardiac reconstruction.\textsuperscript{24,25} It uses only a simple illumination window and avoids longitudinal rebinning (therefore the direction of convolution may not be optimal). The extended cardiac rebinning is evaluated for 16 slice scanners only. Efforts in circular or sequential image reconstructions for medical CT can be found in Refs. 26, 27. This group uses a parallel rebinning T-FDK extension of the original Feldkamp algorithm and achieves promising image quality. However, neither cardiac imaging is considered nor full dose usage is guaranteed due to the truncation of the detector to a virtual rectangular flat panel detector.

Note that EPBP belongs to the class of approximate cone-beam image reconstruction algorithms and could theoretically be outperformed by analytically exact reconstruction algorithms. However, even when considering recent breakthroughs in exact cone-beam reconstruction\textsuperscript{28} these analytical methods are far away from providing a solution to the issues full dose usage and phase-correlated imaging.\textsuperscript{13}

In this paper we specify EPBPStd and EPBPCI. It is organized as follows. A nomenclature section introduces most symbols. Then, the extended parallel backprojection algorithm EPBP is defined. Numerous simulation and measurement results demonstrate the capabilities of EPBP compared to the gold-standard four-slice spiral algorithms 180°MFI and 180°MCI\textsuperscript{29} and ASSR. Due to the focus on medical CT imaging, we restrict our derivations and simulations to cylindrical detector systems that are centered about the focal spot and aligned parallel to the axis of rotation. This is no loss of generality since the respective rebinning equations can be equally well formulated and implemented for other detector types such as flat panel detectors.

\section{Materials and Methods}

The projection or rotation angle is called \( \alpha \). It will be used to parametrize the complete circular, sequential or spiral trajectory. The angle within the fan is given by \( \beta \), the ray's position along the detector's \( z \)-axis is denoted as \( b \). The parallel ray geometry is parametrized by \( \xi \) for the ray's (signed) distance to the origin and \( \vartheta \) for its angle, and we use \( l \) for the parallel detector's longitudinal component. These notations conform to those in Ref. 9.

The cardiac phase as a function of the view angle is denoted as \( c(\alpha) \) and is to be taken modulo 1. It parametrizes the motion with respect to the motion synchronization peaks. In the case of ECG correlation, these peaks are taken as the \( R \) peaks of the ECG. We implicitly assume a correct handling of the modulo properties of \( c \):

\begin{align*}
\lfloor \cdot \rfloor & \quad \text{floor function, yields greatest integer lower or equal} \\
\lceil \cdot \rceil & \quad \text{ceiling function, yields smallest integer greater or equal} \\
\delta(\cdot) & \quad \text{Dirac’s delta function} \\
\text{sgn}(\cdot) & \quad \text{sign function such that } x = |x| \text{sgn } x \\
\alpha_R & \quad \text{ray parametrization for the cylindrical detector} \\
\alpha & \quad \text{projection angle about which the reconstruction is centered} \\
B & \quad \text{detector boundary, } -B \leq b \leq B, \text{ with } B = \frac{MSR_{FD}}{2R_F} \\
\cos \epsilon & \quad \text{length correction factor, } \cos^2 \epsilon = \frac{R_{FD}^2}{b^2 + R_{FD}^2} \\
d_d & \quad \text{table increment per rotation (spherical) or between successive rotations (sequence), } d = (0,0,d) \\
\Phi & \quad \text{fan angle, } \Phi = 2 \sin^{-1}(R_M/R_F), \text{ in our case } 52^\circ \\
\Gamma & \quad \text{cone angle, } \Gamma = 2 \tan^{-1}(B/R_{FD}), \text{ in our case up to } 19.1^\circ \\
M & \quad \text{number of simultaneously acquired slices, in our case } 4,...,256 \\
q_0 & \quad \text{origin of the scan trajectory, } q = (0,0,0) \\
p & \quad \text{sequential or spiral pitch, } p = d/MS \\
p_0(\alpha,\beta,b) & \quad \text{measured projection data} \\
p_1(\delta,\beta,b) & \quad \text{azimuthally rebinned projection data} \\
p_2(\delta,\beta,l) & \quad \text{azimuthally and longitudinally rebinned data} \\
p_3(\delta,\xi,l) & \quad \text{data rebinned to parallel geometry} \\
Q & \quad \text{parameter used to quantify image quality} \\
r & \quad \text{coordinate vector, } r = (x,y,z) \\
R_D & \quad \text{distance from the detector to the center of rotation (z axis); in our case } 470 \text{ mm} \\
R_F & \quad \text{distance from focus to center of rotation (z axis); in our case } 570 \text{ mm} \\
R_{FD} & \quad \text{distance from focus to detector, } R_{FD} = R_F + R_D; \text{ in our case } 1040 \text{ mm} \\
R_M & \quad \text{radius of the field of measurement (FOM); in our case } 250 \text{ mm} \\
S & \quad \text{slice thickness; in our case } 0.75 \text{ mm} \\
s(\alpha) & \quad \text{focus trajectory} \\
w(\delta), \bar{w}(\delta) & \quad 180^\circ\text{-complete weight function and } 180^\circ\text{-normalized weight function} \\
(C/W) & \quad \text{display window center and width of the reconstructed images}
\end{align*}
Cone-beam data were simulated using a dedicated x-ray simulation tool (ImpactSim, VAMP GmbH, Möhrendorf, Germany). The scanner geometry was chosen corresponding to a modern medical CT scanner (Sensation 16, Siemens Medical Solutions, Forchheim, Germany): 1160 projections per rotation and 672 channels per slice were simulated. Phantom definitions were taken from the phantom data base at http://www.imp.uni-erlangen.de/forbexct except for the cardiac motion phantom that is defined in Ref. 30. For simulation, a \( 2 \times 2 \) subsampling was used for the focal spot for the detector; integration time was simulated by subsampling the angular direction with two samples. The starting location of the spiral was \( z_1 = -72 \, \text{mm} - \frac{1}{2} d \) for the thorax phantom and \( z_1 = -72 \, \text{mm} - |d/p| \) for the cardiac phantom. For the circle scan we used \( z = 72 \, \text{mm} \) and \( z = 0 \) for the thorax and the cardiac phantom, respectively. The thorax sequence scan started at \( z = -72 \, \text{mm} \). The angular position for the first view was \( \alpha_1 = 0 \) for all simulations. Poisson noise was added to the simulated data by undoing the log, adding noise, and performing the log again. The number of incident quanta were chosen to get a realistic noise value in the order of 10

Further, a quality factor to relativize variations in resolution and/or image noise and/or pitch value was computed as

\[
Q^2 = \frac{p}{\Delta \rho \Delta z \sigma};
\]

the constant of proportionality was chosen to obtain \( Q = 100\% \) for the gold-standard four-slice scan with \( p=1 \) reconstructed with 180°MFI.

The rationale for the definition of \( Q \) is as follows. For a given dose and a given image reconstruction, the algorithm image noise variance \( \sigma^2 \) is inverse proportional to the size of the \( z \)-resolution element and to the third power of the size of the in-plane resolution element, i.e., \( \Delta \rho^3 \Delta z \sigma^2 = \text{const.} \). Lower values of this constant imply higher image quality, and vice versa. Further, \( \sigma^2 \) is inverse proportional to the applied dose, which, in turn, is proportional to \( 1/p \) (if the tube current is the same for all measurements), thereby dividing the variance by the pitch factor compensates for the differences in dose.

### IV. GEOMETRY

Figure 1(a) shows a view from the operator’s viewpoint onto the CT gantry. The coordinate system is attached to the patient table. The \( z \) axis is defined to be the axis of rotation. The \( y \) axis is defined to point vertically upward (for zero gantry tilt) and the \( x \) axis is given by the orthonormality relation \( x \times y = z \). Moving the table into the gantry yields increasing \( z \) positions.

#### A. Fan beam

The fan-beam geometry of medical CT scanners uses cylindrical detectors of radius \( R_{FD} \). The focal spot rotates at radius \( R_f \) about the isocenter ( \( z \) axis) and the patient table advances by \( d = (0,0,d) \) during one rotation. We use \( \alpha \in \mathbb{R} \) to denote the gantry rotation angle (projection angle), \( \beta \in [-1/2\Phi,1/2\Phi] \) to denote the angle within the fan and \( b \) as the longitudinal detector coordinate. Let \( \mathbf{o} = (0,0,0) \) be the source’s \( z \) position for \( \alpha = 0 \). Then, the source location is parametrized as follows:

\[
\mathbf{s}(\alpha) = R_f \left( \begin{array}{c} \sin \alpha \\ -\cos \alpha \\ 0 \end{array} \right) + \mathbf{d} \frac{\alpha}{2\pi} + \mathbf{o}. \tag{2}
\]

The coordinate vector of the detector \((\alpha, \beta, b)\) is given as

\[
\mathbf{r}(\alpha, \beta, b) = \mathbf{s}(\alpha) + R_{FD} \left( \begin{array}{c} -\sin(\alpha + \beta) \\ \cos(\alpha + \beta) \\ 0 \end{array} \right) + \mathbf{b}. \tag{3}
\]
2B. Parallel beam

For image reconstruction it is convenient to convert the data to in-plane parallel geometry. This means that the z-component of the rays is temporarily ignored, which corresponds to projecting the scan into the x-y plane. The parallel rays are parametrized by their distance \( r \) to the center of rotation and by their angle \( \vartheta \) with respect to the negative y axis. The ray itself is given by the expression \( \vec{x}(\vartheta) + R \vec{y}(\vartheta) \). Thereby, we have introduced the unit vector \( \vec{x} \) pointing from the isocenter to the ray \( (\vartheta, \varphi) \) and the ray’s direction vector \( \vec{y} \):

\[
\vec{x} = (\cos \vartheta, \sin \vartheta), \quad \vec{y} = (-\sin \vartheta, \cos \vartheta).
\]

This definition was chosen to have the central rays for the fan beam \((\beta = 0)\) and for the parallel beam \((\xi = 0)\) coinciding as long as \(\alpha = \vartheta\). The normal form of the ray is the familiar expression \( x \cos \vartheta + y \sin \vartheta - \xi = 0 \).

Please note that
\[
\xi = x \cos \vartheta + y \sin \vartheta \quad \text{and} \quad \eta = y \cos \vartheta - x \sin \vartheta,
\]
allow us to represent a given point \( r = (x, y) \) as \( r = \xi \vec{x} + \eta \vec{y} \) in the rotated \((\xi, \eta)\) coordinate system.

C. Point projection

For 3-D backprojection as it is performed in EPBP, for example, the perspective projection of a point \( r = (x, y, z) \) in the spatial domain from the focal spot \( s(\alpha) \) onto the cylindrical detector is of interest, i.e., we are interested in its \((\beta, b)\) coordinates.

Transaxially, the one-to-one correspondence between a ray’s fan-beam coordinates and its parallel beam coordinates is simple:

\[
\vartheta = \alpha + \beta, \quad \alpha = \vartheta + \arcsin \xi / R_F, \quad \xi = -R_F \sin \beta, \quad \beta = -\arcsin \xi / R_F,
\]

describe the same ray. Therefore, we are free to use the parallel beam coordinates when deriving the point projection coordinates.

For the point \( r \) we can easily state that \( \xi = x \cos \vartheta + y \sin \vartheta \) is its radial detector coordinate. From (5) we immediately find the point’s \( \beta \) coordinate.

The longitudinal detector coordinate must be computed using the intersection theorem. The transaxial distance \( D \) of the respective voxel \( r = (r \cos \varphi, r \sin \varphi, z) \) to the source \( s \) is given as

\[
D^2 = (R_F \sin \alpha - x)^2 + (R_F \cos \alpha + y)^2 = R_F^2 - 2R_F \sin (\alpha - \varphi) + r^2.
\]

Alternatively [cf. Figs. 2 or 1(b)], we can obtain the relation

\[
D = R_F \cos \beta + \eta = \sqrt{R_F^2 - \xi^2 + \eta^2},
\]

with \( \xi \) and \( \eta \) given by (4). Now, compute \( b \) by scaling the axial distance \( z - d(a/2\pi) - o \) of the voxel’s \( z \) coordinate and the source’s \( z \) coordinate from \( D \) to \( R_{FD} \):

\[
b = R_{FD} \left( z - d \frac{\alpha}{2\pi} - o \right).
\]

And, by representing \( \sin \beta \) by the quotient of the opposite leg and the hypotenuse (see Fig. 2), we get another useful representation of \( \xi \):

\[
\xi = -R_F \sin \beta = R_F \frac{x \cos \alpha + y \sin \alpha}{D}.
\]
Below, the quotient of (6a) and (6b) will play a role in determining the optimal direction of convolution along the detector.

V. DEFINITION OF EPBP

EPBP is an approximate 3D cone-beam image reconstruction algorithm concept suitable for various kinds of trajectories; in this paper we restrict our investigations to the circular, sequential, and spiral trajectory. In contrast to ASSR and the theory behind exact image reconstruction that indicated that convolution should rather be performed in the direction of the spiral tangent,13 severe image artifacts were observed, and these algorithms were found to be unacceptable for medical CT. It was the success of ASSR and the theory behind exact image reconstruction that indicated that convolution should rather be performed in the direction of the spiral tangent.13

1. Azimuthal rebinnning

The conversion of fan-beam data $(\alpha, \beta)$ to fan-parallel-beam data $(\vartheta, \beta)$ is called azimuthal rebinnning and corresponds to switching from the independent variable $\alpha$ to the new independent variable $\vartheta$:

$$p_1(\vartheta, \beta, b) = p_0(\alpha, \beta, b).$$

On the rhs, $\alpha = \alpha(\vartheta, \beta)$ is a function of the destination variables $\vartheta$ and $\beta$ according to Eq. (5). For the convection algorithms a length correction factor $\cos \epsilon$ defined by $\cos \epsilon = R_{FD}^2/(b^2 + R_{FD}^2)$ is multiplied to the data during azimuthal rebinnning to account for the obliqueness for the rays with respect to the x-y plane.

2. Longitudinal rebinnning

Until recently, many Feldkamp-type algorithms performed convolution along the direction of constant $b$ just as it is the case with classical $z$-interpolation algorithms. However, severe image artifacts were observed, and these algorithms were found to be unacceptable for medical CT. It was the success of ASSR and the theory behind exact image reconstruction that indicated that convolution should rather be performed in the direction of the spiral tangent.13

For performance reasons it is necessary to perform a row-by-row convolution. Consequently the fan-parallel data have to be converted into an adequate format. This is the reason why longitudinal rebinnning is required. To derive the rebinnning formula, we regard the tangent direction,
3. Radial rebinning

Data are then converted to parallel geometry via radial rebinning,

\[ p_2(\theta, \beta, l) = \frac{ds(\alpha)}{d\alpha}, \]

which is the optimal direction of convolution. The relationship between \( b \) and \( \xi \) when moving an arbitrary object point along the direction \( ds \) and regarding its projection onto the detector can be derived using (6):

\[ \frac{db}{d\xi} = \frac{d(bD)}{d(\xi d)} = \frac{R_{FD}d\xi}{R_F(dx \sin \alpha + dy \sin \alpha)} = \frac{R_{FD}d\xi}{2\pi R_F^2}. \]

In the last step, the components of \( ds \) are inserted. Now, define a new longitudinal variable \( l \) as

\[ b = l + \lambda \xi, \quad \lambda = \frac{db}{d\xi} = \frac{dR_{FD}}{2\pi R_F^2}, \]

such that \( d\lambda d\xi = 0 \) in the direction of \( ds \). Then,

\[ p_2(\theta, \beta, l) = p_1(\theta, \beta, b), \]

with \( b = b(\beta, l) = l + \lambda \xi = l - R_F \sin \beta \) is the longitudinal rebinning that switches to \( l \) as the new independent variable.

The convolution step that precedes the backprojection requires access to all \( \xi \in [-R_M, R_M] \). To allow for longitudinal rebinning without losing parts of the original detector area \( b \in [B, B/2] \) we want to provide \( l \) within the range \( l \in (B + |a| R_M)[-1, 1] \). According to \( b = l + \lambda \xi \) this, however, requires access to \( b \in (B + 2|a| R_M)[-1, 1] \). We provide this area by extending the detector by a simple extrapolation that repeats the outermost detector rows—only then, 100% dose usage can be achieved. This extrapolated area is used exclusively for convolution. Backprojection then respects the original detector area. Figure 4 illustrates the situation. Our procedure is justified because the main weight of the reconstruction kernel lies at its central element that never reaches extrapolated areas. We further have evaluated the influence of extrapolation on the final images by switching extrapolation on and off and found no negative effects.

**B. Convolution**

The remaining steps are convolution and weighted back-projection. Standard convolution kernels for parallel data such as the Shepp–Logan kernel can be used for EPBP. Convolution yields convolved data,

\[ \hat{p}(\theta, \xi, l) = p_3(\theta, \xi, l) * k(\xi). \]

As already mentioned, the convolution exceeds the original detector limits and uses data of the extended detector.

**C. Weighting**

EPBP uses a voxel-dependent weight function \( a \) to perform phase-correlation and \( b \) to normalize the back-projected contributions to 180°.

The number of simultaneously scanned slices ranged from 4 to 256 in our simulations. Since submillimeter slice thicknesses are predominately used in today’s scanners and since our clinical 16-slice scanner allows for 0.75 mm slices, we chose \( S = 0.75 \) mm for all simulations and measurements. These parameters result in the following cone angles (see Table I).

With our implementation of EPBP, three kinds of scan trajectories are possible: the circular scan, the sequence scan, and the spiral trajectory. For the circular scan and for the sequence scan (= multiple circles), it is possible to analytically calculate the \( z \) range that is reconstructable within the FOM. This is best done using the results of Sec. III C. Dropping the \( da/2\pi \) term of (6a) we find that \( |z| \leq DB/R_{FD} \). Now, regard the point \( (R_F \cos \varphi, R_F \sin \varphi) \) at the edge of the FOM. For that point we find \( D^2 = R_F^2 - 2R_F R_M \sin(\alpha - \varphi) + R_M^2 \), whose value obviously changes while the scanner rotates about the point we fixed. If we want the point to be exposed during 360°, one easily finds that the infimum value of \( D \) is \( R_F - R_M \), which is reached whenever the sine evaluates to one. This means \( |z| \leq 1/2MS(1 - R_M/R_F) = 1/2MS(1 - \sin 1/2\Phi) \) or, equivalently, \( d \leq MS(1 - \sin 1/2\Phi) \) and \( p \leq 1 - \sin 1/2\Phi \approx 0.56 \). For data completeness, however, an exposure range of \( \pi + \Phi \) (in fan-beam coordinates) is sufficient. Choosing \( -1/2(\pi + \Phi) - 1/2\pi \leq \varphi \leq 1/2(\pi + \Phi) - 1/2\pi \) yields \( D > R_F \cos 1/2\Phi \). In that case \( |z| \leq 1/2MS \cos 1/2\Phi \) and \( p \approx \cos 1/2\Phi \approx 0.90 \) follows.

In cardiac CT, another degree of freedom is introduced. It is the relation of the patient’s local heart rate \( f_H \) to the scanner’s rotation time \( t_{rot} \). It has been shown (see Refs. 16, 30, and 31) that the decisive parameter is the product \( f_H t_{rot} \). Thus, our cardiac results and image quality are not a function of \( f_H \) and \( t_{rot} \) but rather only a function of \( f_H t_{rot} \). For convenience, we fixed \( t_{rot} = 0.5 \) s while varying the heart rate in a range from 60 to 120 min⁻¹. Conversions to other rotation

**Table I. Cone angles.**

<table>
<thead>
<tr>
<th>( M )</th>
<th>4</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
<th>256</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma )</td>
<td>0.302°</td>
<td>1.21°</td>
<td>2.41°</td>
<td>4.82°</td>
<td>9.63°</td>
<td>19.1°</td>
</tr>
</tbody>
</table>
times $t_{\text{rot}}$, such as the frequently used 0.375 s and the 0.42 s rotation time, can be easily done by solving the relation $f_H t_{\text{rot}} = f_{H'} t'_{\text{rot}}$.

It is of help to introduce the visibility range $V$ of the voxel $r$ of interest: $V(r)$ is the set of all view angles under which $r$ is measured. The visibility range can be computed using the point projection equations of Sec. IV C. For the spiral trajectory, no closed analytical solution can be given and $V$ must be determined numerically. Note that $V$ is not an interval but a union of several disjunct intervals, in general.

Given $V$ phase weighting can be performed by defining a weight function $w(\theta)$ that is zero in the complement of $V$ and that is zero or positive in $V$. Several phase-weighting approaches are found in the literature, ranging from single-phase or partial scan methods (such as the cardio delta approach described in Ref. 16) to bi-phase or even multi-phase approaches. In our case we use the cardio interpolation (CI) algorithm that is a multi-phase approach. It combines multiple allowed data ranges of subsequent motion cycles by using adaptive triangular weights for each data range.$^{16,30}$ The CI’s technique of adaptively optimizing the temporal window width and thereby the temporal resolution led to the

Fig. 5. Gold standard four-slice image reconstruction applied to four-slice data. The cardio data were simulated with $p = 0.375$. Ticks indicate the location of the axial slices and the MPRs (0 HU/300 HU).
Fig. 6. Standard and cardio reconstructions of the thorax phantom with $p = 0.375$ and $f_H = 100 \text{ min}^{-1}$, various algorithms, and various cone angles. Axial slice at $z = 132.75 \text{ mm}$ (0 HU/300 HU).
development of similar approaches that are topic of ongoing research.\textsuperscript{24}

Since phase-correlation can always be formulated as a phase-weighting algorithm EPBP is able to cope with any phase-correlated approach. If one desires to perform a standard reconstruction, a weight function that is constant within \( V \) and zero outside can be used.

A valid weight function has been found as soon as it fulfills the 180° condition,

\[
\sum_{k \in \mathbb{Z}} w(\vartheta + k \pi) > 0, \quad \forall \vartheta;
\]

otherwise no CT images of sufficient quality can be provided.

To avoid data combination artifacts, we further multiply the weight function obtained so far by trapezoidals covering each of the disjunct intervals of \( V \). Thereby, a 7.2° transition range on each side of the trapeze is used in our implementation.

Since \( w \) must be 180° complete, we further normalize the projection weight to get a 180°-normalized weight as follows:

\[
\hat{w}(\vartheta) = \frac{w(\vartheta)}{\sum_{k \in \mathbb{Z}} w(\vartheta + k \pi)}.
\]

The proper normalization of \( \hat{w} \) can be seen from the following identities:

\[
\sum_{k = -\infty}^{\infty} \hat{w}(\vartheta + k \pi) = 1 \quad \text{and} \quad \int_{-\infty}^{\infty} d\vartheta \hat{w}(\vartheta) = \pi.
\]

D. Backprojection

Backprojection means evaluating

\[
f(x,y,z) = \int d\vartheta \hat{p}(\vartheta,\xi,l) \hat{w}(\vartheta),
\]

with

\[
\xi = \xi(x,y,\vartheta) = x \cos \vartheta + y \sin \vartheta, \quad l = b(x,y,z,\alpha) - \lambda \xi, \quad \alpha = \vartheta - \beta = \vartheta + \arcsin \xi/R_F.
\]

This yields the voxel value at \((x, y, z)\). This procedure must be repeated for each voxel location \( r \). Consequently, the weight function implicitly depends on \( r \), too: \( \hat{w}(\vartheta) = \hat{w}(r,\vartheta) \). Note that phase-correlation and the full dose usage both are hidden in the definition of the weight function which, in turn, depends on the voxel-dependent visibility range \( V \).

VI. SIMULATIONS AND MEASUREMENTS

To evaluate our new approach and to benchmark it against the other algorithms, various simulations were performed. We restricted ourselves to the geometry of a medical CT
scanner with $R_f = 570$ mm, with 1160 views per rotation and a 52° fan angle that is transaxially covered by 672 detector elements.

Summarizing, the following parameter set has been used for evaluation:

- $M \in \{4, 16, 32, 64, 128, 256\}$
- $p \in \{0.375, 0.5, 0.9, 1.0, 1.5\}$
- $f_H \in \{60, 80, 100, 120\}$ min$^{-1}$
- trajectory $\in \{\text{circle, sequence, spiral}\}$
- algorithms $\in \{180°\text{MFI}, 180°\text{MCI}, \text{ASSRStd}, \text{ASSRCI}, \text{EPBPCI}, \text{EPBPStd}\}$

some of these combinations are shown in Sec. VII. We apologize for not being able to show a comprehensive overview of all parameter combinations simulated and evaluated. The limited space of the paper does not allow us to do so. Nevertheless, we will also discuss some results that are not reproduced here.

To quantify our results we have further computed the in-plane point spread function (PSF), the slice sensitivity profiles (SSP) that constitute the axial PSF and image noise.

### VII. RESULTS

The four-slice algorithms 180°MFI and 180°MCI applied to four slice data have been chosen as the gold standard in that paper; the other algorithms must compare to that standard. Gold standard images are shown in Fig. 5. Please note that only the 180°MFI reconstruction with $p = 1$ (top right) is used to normalize the $Q$ factor of Eq. (1). Therefore it is the only figure where $Q$ exactly equals 100%.

#### A. Results for the spiral trajectory

To give a first overview over the image quality that can be achieved with the variety of algorithms a compilation of axial images is presented in Fig. 6. It shows standard and cardio reconstructions of the thorax phantom reconstructed at $z = 132.75$ mm for $p = 0.375$. The gold-standard 180°MFI achieves high image quality in the case of four slices. With 16 slices, however, the humerus starts suffering from blurring artifacts. For higher pitch reconstructions (not shown here), geometrical distortions become visible. The ASSR reconstruction of the same 16 slice data shows no such artifacts. When moving to 128 slices or more (not shown) the ASSR’s image quality becomes severely degraded, as expected. EPBP, in contrast, results in much better images. There are no geometrical distortions and no artifacts produced by the

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**Image Note:**
- **Fig. 8.** Spiral scan with $M = 256$ slices of the thorax phantom. Standard reconstruction. Ticks indicate the location of the axial slices and the MPRs ($0 \text{ HU}/300 \text{ HU}$).
high contrast bone structures for EPBP. Evidently, the low pitch EPBP reconstructions appear to be equivalent to the gold-standard reconstructions.

For the cardiac images of Fig. 6, advantages of ASSRCI over 180° MCI at 16 slices are not apparent. With $M = 32$ slices, clear improvements of EPBPCI over ASSRCI can be seen. Even for as many as 256 slices the EPBPCI does not suffer from distortion artifacts. However, slight artifacts in the tissue-equivalent parts of the phantom are visible. In all cases, the artifact behavior varies with the heart rate and with the chosen reconstruction phase (not shown). This is because the allowed data ranges are a function of the heart rate and reconstruction phase.

To give a more comprehensive overview, MPR displays are needed in addition to the axial images. The following sequence of figures shows the planes $x = 0$ mm (sagittal), $y = 0$ mm (coronal), and $z = 0$ mm (axial) of the reconstructed thorax phantom. The remaining fourth quadrant is used to display the quantitative parameters for image noise, spatial resolution, and the quality index $Q$.

Figure 7 shows ASSR and EPBP standard reconstructions of the thorax phantom for a 64-slice scanner and two pitch values. Axial as well as MPR views are presented. The phantom’s ribs are made up of inclined hollow cylinder segments and can be seen in the axial and in the coronal images. They are of special interest since they tend to easily produce artifacts. For example, artifacts around the ribs are visible with ASSRStd for the $p = 1.0$ scan. For EPBPStd, no rib artifacts

![Fig. 9. Spiral scan ($p = 0.375$) with $f_{\mu} = 120$ min$^{-1}$ of the thorax phantom with an increasing number of simultaneously scanned slices. Cardio reconstruction. Ticks indicate the location of the axial slices and the MPRs (0 HU/300 HU).](image-url)
are visible. The sagittal images show the sternum, the heart, and the spine. The vertebra, and similarly the transitions between the thorax and the shoulder segments, tend to produce high-frequency streak-like artifacts that are oriented horizontally. These are of little concern since realistic scans include scatter and physical objects that do not exhibit mathematically perfect alignment. A comparison to the 256-slice reconstructions of Fig. 8 shows still excellent images for

**Fig. 10.** Reconstructions of the cardiac motion phantom scanned with $M = 64$ slices at $60 \text{ min}^{-1}$ with various algorithms (0 HU/500 HU).
EPBP whereas the ribs in the ASSR reconstructions are severely degraded. Again, the low pitch images are of better quality than the high pitch images. Regarding the quantitative values for these two figures, we find that image noise is increased with ASSR and the $Q$ is below 100%. EPBP achieves remarkably high $Q$, which is mainly due to the lower image noise and due to the improved $z$ resolution.

The next series of figures shows phase-correlated recon-

Fig. 11. Reconstructions of the cardiac motion phantom scanned with $M = 64$ slices at 100 min$^{-1}$ with various algorithms (0 HU/500 HU).
structions of the thorax phantom (which is stationary) and of the cardiac motion phantom. Figures of merit given for the thorax phantom apply to the reconstructions of the motion phantom as well: the scan and reconstruction parameters are identical in both cases.

The CI reconstructions of Fig. 9 have been synchronized using a heart rate of 120 min$^{-1}$. ASSRCI shows significant blurring in regions off the rotation axis. Artifacts mainly manifest in the inclined ribs but also the sagittal display shows blurring. The vertebrae are not delineated very well and the MPRs appear smoothed. Even the humerus is significantly degraded. This is confirmed by the FWHM of the SSP that lies at 1.4 mm for the 64-slice scanner. In contrast to the advanced single-slice rebinning reconstruction, EPBP achieves good delineation of all structures, even for those that are far away from the rotation center. This applies for all cone angles simulated, i.e., up to 256 slices.

The cardiac motion phantom is shown with various algorithms and three reconstruction phases in Figs. 10 and 11 for the heart rates 60 and 100 min$^{-1}$, respectively. We used the spiral 64-slice scanner with $\rho = 0.375$ to produce these images. The top row is reconstructed using the four-slice algorithms, the middle row is ASSR, and the bottom row is EPBP. The first column shows a standard reconstruction, for comparison, whereas the remaining three columns depict the motion phantom in the slow (0%), medium (30%), and high motion (50%) phase, respectively.

Inspecting the axial images, we find only little variations between the three algorithms. Here, EPBP tends to show slightly less motion artifacts. This can be seen especially in the slow motion phase (0% of S–S) of the 60 min$^{-1}$ and of the 100 min$^{-1}$ reconstructions (Figs. 10 and 11).

The 120 min$^{-1}$ case that is not shown here exhibits motion artifacts for all algorithms and reconstructed phases. This behavior, which has already been thoroughly described and analyzed in Refs. 16, 30, 32, is due to the resonance behavior of the heart motion and the scanner rotation: both take place at a frequency of 120 min$^{-1}$. At that frequency, the duration of the slow motion phase (20% of the cardiac cycle) is as low as 100 ms. The cardio interpolation is not able to improve its temporal resolution by collecting data from adjacent motion cycles since these are in resonance with the rotation angle. Therefore, CI’s temporal resolution is only 250 ms (half of the rotation time) and motion will even be seen in the slow motion phase. The same behavior is expected for the 60 min$^{-1}$ case, which is also in resonance with the scanner. Here, however, the slow motion phase lasts 200 ms, which is only insignificantly lower than the 250 ms temporal resolution, and motion artifacts are not apparent (see Fig. 10). For the 80 min$^{-1}$ (not shown) and 100 min$^{-1}$ reconstructions there is no resonance behavior and the algorithm’s temporal resolution is high enough to freeze motion in the slow motion phase completely. One possibility to avoid resonance cases is adjusting the rotation time accord-
ing to the patient’s (expected) heart rate, as recommended in Refs. 16, 30, 32. Some manufacturers provide respective product implementations.

More significant advantages of EPBP over the other algorithms can be found in the sagittal and coronar MPRs that show the 2 mm objects. Here, EPBPCI exhibits far less blurring in the $z$ direction than 180°MCI and ASSRCI.

B. Results for the sequence trajectory

We have used EPBPStd to perform sequence reconstructions of the thorax phantom for two different pitch values and for $M$ ranging from 16 to 256. Figure 12 shows examples with 32 and 256 slices. In all cases, image quality is satisfactory. Images are slightly better for the $p = 0.375$ scan and artifacts in the MPR tend to increase for an increasing number of slices. Compared to the corresponding spiral scans and EPBPStd reconstructions (e.g., the $p = 0.375$ reconstruction of Fig. 7), we find the sequence trajectory to be competitive regarding image quality.

In the sequence mode, EPBPCI combines allowed data segments in the order of 1/$p$ adjacent rotations (depending on the $z$ position) to become phase selective and to achieve a high temporal resolution. Since the sequence cardio scan mode seems to have no practical importance, we do not show respective results, however.

C. Results for the circular trajectory

Figures 13 and 14 show EPBPStd and EPBPCI reconstructions for circle scans of the thorax phantom and of cardio motion phantom, respectively. Since for EPBPCI the number of rotations has been chosen to be 1/0.375 the same amount of data overlap and hence the same temporal resolution as in the spiral case can be expected.

From Fig. 14 we can see that the circle scan EPBP reconstruction behaves very well regarding the motion artifacts and the sharpness of the structures in the $z$ direction. It is this trajectory that has the potential to perform dynamic CT studies of the heart in the near future.

D. Patient data

To demonstrate EPBP’s capabilities of processing clinical data reconstructions of standard 12-slice CT data have been performed. Figure 15 shows axial slices and MPRs of one patient reconstructed with ASSR and EPBP in both the standard and cardio interpolation modes. The latter uses kymogram data for synchronization. To illustrate the phase selectivity, two phases have been chosen for reconstruction: 0% and 50% of K–K.

EPBP achieves very high image quality with these medical data. The standard reconstructions are blurred due to the heart motion. Apparently, ASSRStd suffers from smooth
horizontal streaks that are not visible in the EPBPSstd reconstruction. The CI reconstructions are of high quality for both reconstruction algorithms. Due to EPBP's higher spatial resolution, image noise is slightly increased compared to ASSR. This confirms our observations of the phantom study and can be compensated for by choosing smoother reconstruction kernels.

Similar results are obtained with the patient in Fig. 16: EPBP achieves excellent image quality that is of the same quality as today's state-of-the-art cardiac reconstructions. The reader should note that this good performance of EPBP for as few as 12 slices is far from evident. Since EPBP is designed to be used with scanners with far more slices and since EPBP performs complicated oblique integrations on the detector area, one might expect discretization and border effects to become dominating for small detectors. Fortunately, this is not the case.

VIII. DISCUSSION

The extended parallel backprojection is a generalized algorithm that combines and improves the capabilities of several reconstruction approaches. EPBP is able to reconstruct data from circular, sequential, and spiral trajectories in both the standard mode and the phase-correlated reconstruction mode.

It has been shown that the proposed algorithm achieves very high image quality for a huge range of parameters. EPBP performance has been demonstrated for scanners with as many as 256 slices, for pitch values ranging from 0.375 to 1.5, and for heart rates between 52 and 120 min\(^{-1}\). Further on, EPBP copes well with data stemming from today's 12- or 16-slice algorithms.

Our quantitative results indicate that EPBP makes better use of the available data than ASSR: it achieves better resolution and lower image noise. This may be mainly attributed to the fact that the degree of approximation used with EPBP is less than for ASSR. Thereby, EPBP is even comparable to the gold standard 180°MFI and 180°MCI, respectively. However, the fact that EPBP's \(Q\) happens to be larger than that of 180°MFI must be taken with care: the quality factor does not take into account the complete shape of the point spread function; it rather accounts for one figure of merit, namely the full width at half maximum.

As a general result, we further observed that reconstructions of low pitch data result in better image quality and less artifacts than reconstructions of high pitch data. Obviously due to the data redundancy (scan overlap) cone-beam artifacts are somehow averaged and appear less dominant.

For cardiac reconstructions the same temporal resolution and the same motion artifact behavior as known from the older algorithms has been observed with EPBP. This applies even for the cardiac circle scan where no profound reports can be found in the literature, up to now. We have seen that the cardiac circle scan is comparable to the cardiac spiral scan except for few cases with increased motion artifacts. Our statements are valid for as many as 256 simultaneously scanned slices and probably more. Here, the recommenda-
FIG. 15. Patient example, 52 min$^{-1}$ (0 HU/1000 HU).
Fig. 16. Patient example, 70 min$^{-1}$ (0 HU/700 HU).
tion to use variable rotation times (as a function of the heart rate) can be put forward again. For the cardiac sequence scan one might even control the interscan delay as a function of heart rate to optimize the position of the allowed data windows.

Currently, the only drawback is the reduction of reconstruction speed by a factor 2–5 (depending on the pitch value) compared to the z-interpolation approaches. Since our implementation is not yet optimized and since computational speed will increase according to Moore’s law we find that reconstruction speed is not really an obstacle.

Altogether, our results indicate that EPBP is an image reconstruction algorithm that handles the current and future demands of medical CT. It allows for arbitrary pitch spiral and sequential scans with or without phase correlation. Regardless of the cone-angle, EPBP ensures 100% dose usage and shows only minor cone-beam artifacts. Of particular interest is the cardiac circle reconstruction that even allows for dynamic studies of the heart—especially when considering that commercial 64-slice CT systems have recently become available.

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