Abstract—Medical CT aims at faster rotation speeds and a higher number of simultaneously acquired slices. These efforts are pushed further by cardiac CT (4D reconstruction) which is currently the most prominent special-purpose application in CT. Currently, 16-slice scanners and rotation times of below 0.5 s are state of the art. Scanners with far more slices have already been announced. Among those developments are prototypes with up to 256 slices.

The basic requirements for image reconstruction are the support of circular, sequence and spiral scans. Arbitrary pitch selection is of high importance. Further, the complete area of the detector is to be exposed and each measured ray should contribute to the image to achieve optimized dose usage.

Only approximate reconstruction approaches have the potential to handle all these requirements. Currently, the only known phase-correlated true cone-beam approach is an extension to the Advanced Single-Slice Rebinning (ASSR) algorithm [1]. However, this generalized approach ASSR CI is limited to 32 slices.

We have therefore developed a new approximate Feldkamp-type algorithm, the extended parallel backprojection (EPBP). Its main features are a phase-weighted backprojection and a voxel-by-voxel 180° normalization. The first feature ensures 3D and 4D capabilities with one and the same algorithm, the second ensures 100% detector usage (each ray counts!). The algorithm is evaluated using simulated circular, sequential and spiral data of a thorax phantom and of a cardiac motion phantom and measured patient data for scanners with up to 256 slices.

The standard reconstructions (EPBP Std) are of excellent quality even for as many as 256 slices regardless of the scan trajectory. The cardiac reconstructions (EPBP CI) are of high quality as well and show no significant deterioration of objects even far off the center of rotation. Since EPBP CI uses the cardio interpolation (CI) phase weighting the temporal resolution is equivalent to that of the well known single-slice and multi-slice cardiac approaches 180°CI, 180°MCI, and ASSR CI, respectively, and lies in the order of 50 ms to 100 ms for rotation times between 0.375 s and 0.5 s.

I. INTRODUCTION

Computed tomography is currently evolving faster than ever. Increased spatial resolution, decreased scan time, increased temporal resolution, decreased patient dose, and increased volume coverage are some of the important trends to mention. The near future will increase the number of simultaneously scanned slices to 32, 64 and even more. Slice thickness, and thereby spatial resolution, will continue to decrease to further improve diagnostic accuracy. At the same time, dose utilization will increase to keep the effective patient dose at an acceptable level: dose modulation techniques, automatic exposure control and improved detector materials will help to do so [2]. Besides improved spatial resolution, improved contrast resolution and low patient dose there is interest in highest temporal resolution to allow imaging the heart. This is done using short rotation times combined with dedicated phase-correlated reconstruction algorithms as they are known since the mid-nineties [3].

Since today’s cone-beam reconstruction algorithms do not ensure 100% dose usage or they do not work for arbitrary spiral pitch or they are not capable of phase-correlated imaging at wider cone angles we propose EPBP, a new approximate Feldkamp-type cone-beam reconstruction that comprises all these capabilities.

EPBP is dedicated to reconstruct circular, sequence (= multiple circles) and spiral data of a cone-beam CT scanner with cylindrical detectors. In this paper, we will outline the EPBP Std and the EPBP CI algorithm and give some descriptive examples.

Fig. 1. Spiral thorax scan with 256 × 0.75 mm collimation and d = 72 mm table increment per rotation. A heart rate of 120 min−1 was assumed for EPBP CI reconstruction which results in about 25% data in the allowed phase intervals. (0/500)
II. Simulations

To evaluate our new approach we have simulated spiral cone-beam data corresponding to the in-plane geometry of a typical medical CT scanner (1160 projections per rotation, 672 detector channels per detector row, and a fan angle $\Phi = 52^\circ$) using a dedicated x-ray simulation tool (ImpactSim, VAMP GmbH, Mührendorf, Germany). Two phantoms have been simulated: the thorax phantom described in the phantom data base http://www.imp.uni-erlangen.de/forbild and the cardiac motion phantom described in [4].

The simulated scan protocol uses 0.42 s rotation time (143 rpm), $M \times 0.75$ mm collimation with $M = 2^{m}$ simultaneously scanned slices where $m = 4, \ldots, 8$. The pitch value is defined as $p = \frac{d}{MS}$ for spiral and sequence trajectories and as $p = \frac{1}{N_{\text{rot}}}$ for circle scans where $N_{\text{rot}}$ is the (not necessarily integer) number of rotations carried out. The table increment for sequence scans is set to zero.

For the results here, we chose $p = 0.375$ which means a 2.7-fold data redundancy. This is a typical value used for phase-correlated CT image reconstruction. Evaluation for other pitch values is carried out in reference [5].

III. Geometry

A. Scan Geometry

The scan geometry assumed here is a fan-beam geometry with cylindrical detectors and a spiral focus trajectory. The EPBP approach is based on a rebinning to parallel geometry. Other geometries, such as flat detectors, can be easily incorporated by modifying the corresponding rebinning and transform equations. Note that in the limit of zero table increment, the spiral reduces to a circular trajectory. EPBP copes with circular and sequence data as well as it does with spiral data.

The spiral source trajectory is parameterized by the view angle $\alpha$ as

$$
s(\alpha) = R_F \left( \begin{array}{c} -\sin \alpha \\ -\cos \alpha \\ 0 \end{array} \right) + d \left( \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right) \frac{\alpha}{2\pi}. \tag{1}$$

$R_F$ denotes the radius of the focal spot trajectory and $d$ denotes the table increment per rotation. For sequence scans, the term $\alpha/2\pi$ is to be replaced by $\lfloor \alpha/2\pi \rfloor$. For circle scans $d = 0$.

The coordinate vector of the detector element $(\alpha, \beta, b)$ is given as

$$\mathbf{r}(\alpha, \beta, b) = s(\alpha) + R_{\text{FD}} \left( \begin{array}{c} -\sin(\alpha + \beta) \\ \cos(\alpha + \beta) \\ 0 \end{array} \right) + b \left( \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right);$$

$\beta$ and $b$ denote the transaxial and longitudinal detector components, respectively.

To rebin the transaxial components of the cone-beam data to parallel geometry we parameterize a ray by its distance $\xi$ to the axis of rotation and by its angle $\vartheta$ with respect to the negative $y$-axis. The normal form of the ray’s $x$-$y$-components is given as $x \cos \vartheta + y \sin \vartheta - \xi = 0$. This definition was chosen to have the central rays for fan-beam ($\beta = 0$) and for parallel beam ($\xi = 0$) coinciding for $\alpha = \vartheta$.

The relation between a ray in fan-beam coordinates $(\alpha, \beta)$ and a parallel-beam ray $(\vartheta, \xi)$ is the familiar transform

$$\vartheta = \alpha + \beta \quad \text{and} \quad \alpha = \vartheta + \arcsin \xi / R_F \quad \beta = -\arcsin \xi / R_F. \tag{2}$$

B. Point Projection

For backprojection, we need to know the detector coordinates $(\xi, b)$ that result from projecting the point $(x, y, z)$ from $s(\alpha)$ onto the cylindrical detector. The radial coordinate is given as $\xi = x \cos \vartheta + y \sin \vartheta$. The longitudinal detector coordinate can be computed using the intersection theorem. The transaxial distance of the respective voxel to
the source is given as
\[ D^2 = (R_F \sin \alpha - x)^2 + (R_F \cos \alpha + y)^2 \]
\[ = R_F^2 - 2R_F r \sin(\alpha - \varphi) + r^2 \]
with \((x, y) = (r \cos \varphi, r \sin \varphi)\) or, equivalently,
\[ D = R_F \cos \beta + \eta = \sqrt{R_F^2 - \xi^2 + \eta} \]
with \(\eta = y \cos \vartheta - x \sin \vartheta\). Now, find \(b\) by scaling the axial distance \(z - \frac{d\alpha}{2\pi}\) from \(D\) to \(R_F D\):
\[ b = \frac{R_F D}{D} \left( z - \frac{d\alpha}{2\pi} \right). \] (3a)

And we find another representation of \(\xi\):
\[ \xi = -R_F \sin \beta = R_F \frac{x \cos \alpha + y \sin \alpha}{D}. \] (3b)

IV. Reconstruction

The extended parallel backprojection algorithm consists of the following five steps:
- azimuthal rebinning: \(p(\alpha, \beta, b) \rightarrow p(\vartheta, \beta, b)\),
- longitudinal rebinning: \(p(\vartheta, \beta, b) \rightarrow p(\vartheta, \beta, l)\),
- radial rebinning: \(p(\vartheta, \beta, l) \rightarrow p(\vartheta, \xi, l)\),
- convolution: \(p(\vartheta, \xi, l) \rightarrow \hat{p}(\vartheta, \xi, l)\),
- weighting and backprojection: \(\hat{p}(\vartheta, \xi, l) \rightarrow f(x, y, z)\).

A. Azimuthal Rebinning

The original projection data \(p(\alpha, \beta, b)\) are converted from fan–beam to fan–parallel geometry using (2) as follows:

\[ p(\vartheta, \beta, b) = p(\alpha, \beta, b) \quad \text{with} \quad \alpha = \vartheta - \beta. \]

B. Longitudinal Rebinning

Convolving spiral data in the detector row direction (constant \(b\)) yields severe cone–beam artifacts. As indicated by ASSR [1], SMPR [6], and exact cone–beam reconstruction [7] the optimal direction of convolution is the tangent \(ds(\alpha)/d\alpha\). To align the fan–parallel detector rows with the optimal direction of convolution a longitudinal rebinning is required. Therefore, we are interested in the relationship between \(b\) and \(\xi\) when moving along \(ds\). Using (3) one finds

\[ \frac{db}{d\xi} = \frac{d(bD)}{d(\xi D)} = \frac{R_F d}{R_F d \cos \alpha + d\sin \alpha} = \frac{R_F d}{2\pi R_F^2}; \]

in the last step (1) was used to insert the components of \(ds\). Now, we define a new longitudinal variable \(l\) as

\[ b = l + \lambda \xi \quad \text{with} \quad \lambda = \frac{db}{d\xi} = \frac{dR_F}{2\pi R_F^2}; \]

such that \(dl/d\xi = 0\) in the direction of \(ds\). Then, do the longitudinal rebinning
\[ p(\vartheta, \beta, l) = p(\vartheta, \beta, b) \quad \text{with} \quad b = l + \lambda \xi = l - \lambda R_F \sin \beta \]

C. Radial Rebinning

The radial rebinning converts to equidistant parallel coordinates. We use (2) to find
\[ p(\vartheta, \xi, l) = p(\vartheta, \beta, l) \quad \text{with} \quad \beta = -\arcsin(D/R_F). \]
D. Convolution

Now, convolution of the detector rows is performed using a standard convolution kernel \( k(\xi) \), as, for example, the Shepp–Logan kernel. \( \hat{p}(\vartheta, \xi, l) = p(\vartheta, \xi, l) * k(\xi) \) yields the convolved data \( \hat{p} \) needed for backprojection.

E. Weighting and Backprojection

In this step, we regard the backprojection of a fixed voxel, say one located at \( r = (x, y, z) \). Let \( V \) denote the set of view angles \( \vartheta \) under which \( r \) is measured.

Assume a temporal window \( T \) that comprises all \( \vartheta \) that correspond to allowed data. For the standard reconstruction EPBP Std, all data acquired are allowed data and therefore \( T = \mathbb{R} \). For the reconstruction of cardiac data, \( T \) can be defined by specifying a cardiac motion phase \( \varepsilon \) that counts relative to some synchronization peaks and a phase width \( 0 < \Delta \varepsilon \leq 1; T \) will then consist of a set of disjunct intervals. EPBP CI chooses \( \Delta \varepsilon \) as small as allowed by the completeness condition (see below). Other definitions may include absolute timing information or the restriction to only one temporal interval of length \( \pi \) (single–phase reconstruction), or two intervals (bi–phase reconstruction).

Regardless of what convention is used to define \( T \), the intersection \( I = V \cap T \), that comprises all views to be used, must be \( 180^\circ \)–complete:

\[
\bigcup_k (I + k \pi) = \mathbb{R}.
\]

Now, for each voxel \( r \) assume a weighting function \( w(r, \vartheta) \) whose support equals \( I \), i.e. \( w(r, \mathbb{R} \setminus I) = \{0\} \), and \( \sum_k w(r, \vartheta + k \pi) \neq 0 \). The last condition can easily be achieved by using positive weights on \( I \) only. For EPBP CI we use a multi–triangular weight function: triangle functions located on each of \( I \)'s disjunct intervals.

By normalizing \( w \) as

\[
\bar{w}(r, \vartheta) = \frac{w(r, \vartheta)}{\sum_k w(r, \vartheta + k \pi)}
\]

we achieve

\[
\sum_k \bar{w}(r, \vartheta + k \pi) = 1 \quad \text{and} \quad \int d\vartheta \bar{w}(r, \vartheta) = \pi.
\]

Backprojection

\[
f(x, y, z) = \int d\vartheta \hat{p}(\vartheta, \xi, l) \bar{w}(r, \vartheta)
\]

with

\[
\xi = \xi(x, y, \vartheta) = x \cos \vartheta + y \sin \vartheta
\]

\[
\alpha = \vartheta - \beta = \vartheta + \arcsin \xi/R_F
\]

\[
l = b(x, y, z, \alpha) - \lambda \xi
\]

then yields the desired voxel value at \( r = (x, y, z) \).

V. Results

Figure 1 shows that image quality of the thorax phantom is excellent with EPBP, even for as many as 256 slices. As indicated by the ribs, ASSR (which is in fact designed for up to about only 60 slices [1]) cannot cope with this large cone–angle; the same applies to the highly related AMPR algorithm defined in reference [8].

![Cardio EPBP reconstructions of 256–slice scans of the thorax phantom for various trajectories. (0/300)](image)

Fig. 4. Cardio EPBP reconstructions of 256–slice scans of the thorax phantom for various trajectories. (0/300)

Considering that EPBP CI uses only a fraction of the data available (here, roughly 25%), depending on the local heart rate and on the reconstruction position, the images are almost as good as the EPBP Std reconstructions, apart from the increased image noise. The only exception is a slight variation in the reconstructed density close to the vertebrae.

Figure 2 shows reconstructions of the cardiac motion phantom for a spiral 256–slice scan. Since the field of view shows only the central parts of the patient, the in–plane images of the ASSR approach are acceptable even for the 256–slice scanner. However, with 256–slice ASSR the multiplanar reformations (MPRs) tend to be blurred in the \( z \)–direction and full width at half maximum FWHMz of the slice sensitivity profile is increased significantly whereas the in–plane resolution FWHMxy is the same as for the 16–slice case.

EPBP, in contrast, behaves very well for all simulated scanners (16, 32, 64, 128 and 256 slices). Spatial resolution is slightly higher than for the single–slice rebinning algorithms. Image noise increases for EPBP CI due to the phase–weighting. This observation is valid for all the other simulated geometries and heart rates (we have looked into \( f_H = 40 \text{ min}^{-1}, \ldots, 140 \text{ min}^{-1} \)). EPBP generally behaves equal to or better than ASSR. Examples for EPBP Std and CI reconstructions with \( M = 256 \) and various kinds of trajectories are given in figures 3 and 4.
Finally, figure 5 gives an example of reconstructed patient data. The data shown are correlated to the patient motion function, the so-called kymogram, which can directly be derived from the acquired raw data [9]. The standard reconstructions of ASSR Std and EPBP Std are comparable due to the low number of slices; the phase–correlated EPBP CI images are of high image quality and correspond to the gold–standard in cardiac CT imaging.

VI. Discussion

The extended parallel backprojection appears to be adequate for medical CT image reconstruction in all respects. EPBP image quality is equivalent to existing 4– or 16–slice standard and cardiac algorithms for a wide range of simultaneously scanned slices. Even data with $M = 256$ slices yields excellent image quality. For standard reconstructions this is not surprising since EPBP Std is similar to other Feldkamp algorithms (as long as these perform convolution along the tangent direction). For wide cone angle cardiac data, where no other phase–correlated cone–beam algorithm is available yet, EPBP CI performs very well even for objects far off the isocenter (ribs in figure 1).

Feldkamp–type algorithms are superior to ASSR [1], AMPR [8], or SMPR [6] for large $M$. Wide cone angle cardiac CT is currently only possible with EPBP. Its unique weighting strategy that assigns individual data ranges to each voxel, ensures 100% data usage and thus the maximum dose utilization possible. The future of medical sequential and spiral CT will certainly include the idea of phase–correlated/phase–weighted 3D backprojection of EPBP–type, or modifications thereof.

REFERENCES