Empirical cupping correction: A first-order raw data precorrection for cone-beam computed tomography

Marc Kachelrieß, a) Katia Sourbelle, and Willi A. Kalender
Institute of Medical Physics, University of Erlangen-Nürnberg, Henkestraße 91, D-91052 Erlangen Germany

(Received 5 December 2005; revised 13 February 2006; accepted for publication 21 February 2006; published 19 April 2006)

We propose an empirical cupping correction (ECC) algorithm to correct for CT cupping artifacts that are induced by nonlinealities in the projection data. The method is raw data based, empirical, and requires neither knowledge of the x-ray spectrum nor of the attenuation coefficients. It aims at linearizing the attenuation data using a precorrection function of polynomial form. The coefficients of the polynomial are determined once using a calibration scan of a homogeneous phantom. Computing the coefficients is done in image domain by fitting a series of basis images to a template image. The template image is obtained directly from the uncorrected phantom image and no assumptions on the phantom size or of its positioning are made. Raw data are precorrected by passing them through the once-determined polynomial. As an example we demonstrate how ECC can be used to perform water precorrection for an in vivo micro-CT scanner (TomoScope 30 s, VAMP GmbH, Erlangen, Germany). For this particular case, practical considerations regarding the definition of the template image are given. ECC strives to remove the cupping artifacts and to obtain well-calibrated CT values. Although ECC is a first-order correction and cannot compete with iterative higher-order beam hardening or scatter correction algorithms, our in vivo mouse images show a significant reduction of bone-induced artifacts as well. A combination of ECC with analytical techniques yielding a hybrid cupping correction method is possible and allows for channel-dependent correction functions. © 2006 American Association of Physicists in Medicine. [DOI: 10.1118/1.2188076]

Key words: flat-panel detector CT, C-arm CT, micro-CT, artifacts, image quality

I. INTRODUCTION

Due to beam polychromacity in CT, the energy dependence of the attenuation coefficients, and scatter, the log-attenuation is a nonlinear function of the object density. Image reconstruction typically assumes linearity and, unless the nonlinearity is not properly compensated for, cupping artifacts show up in the final images.

Many methods have been proposed to correct the cupping artifact. Usually preprocessing is applied to the projections before reconstruction1–4 to linearize the attenuation data. This preprocessing is based on empirical functions determined with dedicated calibration phantoms or it is based on the a priori knowledge of the spectrum and attenuation coefficients. One would classify these approaches as first-order correction techniques since they do not require knowledge of the object. Second- or higher-order beam-hardening correction methods exist as well5–7 but they are iterative, slow, and therefore not in routine use. They iterate between raw data domain and image domain. In contrast to the first-order corrections the higher-order methods strive to completely remove more sophisticated artifacts such as dark streaks between bones or metal since they can correct for more than one material present.

In this work, we want to present a simple empirical cupping correction (ECC) algorithm. In contrast to other methods, ECC does not require knowledge of the spectrum and the attenuation coefficients, nor does it require to exactly know the calibration phantom shape, size, and position. Therefore, it has significant advantages over the other existing approaches that actually rely on this information.

ECC aims at linearizing the measurement using an optimized precorrection function that can be decomposed into a linear combination of functions of the original, uncorrected raw data. In our examples we use linear combinations of monomials and, therefore, data precorrection means passing each measured datum through a polynomial. The fit that determines the coefficient vector is performed once (for each combination of tube voltage and prefiltration) by measuring an arbitrarily shaped but homogeneous test object and solving a linear system.

ECC is demonstrated using data acquired with a dedicated small animal imaging in vivo micro-CT scanner. ECC can be used for other CT modalities as well. Our experience also covers the correction of clinical CT data and of C-arm CT data. In general, ECC seems of interest to flat panel detector-based CT systems where scatter is a dominant source of cupping besides beam hardening.3

II. METHOD

Let \( q(L) \) be the measured CT raw data value (log-attenuation) and let \( p(L) \) be the desired corrected raw data value that corresponds to the line of integration (or ray, or detector channel number) \( L \). For convenience we will drop the index \( L \) in the following. We define

\[ q(L) = \sum_{l} a_l p(l) \]

where \( a_l \) is the coefficient of the monomial \( p(l) \) and \( L = \sum_{l} l a_l \) the total integral over the detector. The coefficients \( a_l \) are determined by minimizing the mean squared error between the measured and corrected data.

The linear system to solve is

\[ \mathbf{A} \mathbf{a} = \mathbf{b} \]

where \( \mathbf{A} \) is an \( M \times N \) matrix containing the values \( a_l \) and \( \mathbf{b} \) contains the values \( q(L) \). The solution vector \( \mathbf{a} \) is then

\[ \mathbf{a} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b} \]

Note that the matrix \( \mathbf{A}^T \mathbf{A} \) is symmetric and positive definite, and thus invertible. The solution is computed with a standard least-squares solver.

The above formulation can be extended to higher-order corrections by including higher-order terms in the monomials. For example, a second-order correction would include terms like \( p(2L) \) and \( p(3L) \). The coefficients \( a_l \) are determined by minimizing the mean squared error between the measured and corrected data.

The linear system to solve is

\[ \mathbf{A} \mathbf{a} = \mathbf{b} \]

where \( \mathbf{A} \) is an \( M \times N \) matrix containing the values \( a_l \) and \( \mathbf{b} \) contains the values \( q(L) \). The solution vector \( \mathbf{a} \) is then

\[ \mathbf{a} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b} \]

Note that the matrix \( \mathbf{A}^T \mathbf{A} \) is symmetric and positive definite, and thus invertible. The solution is computed with a standard least-squares solver.
\[ p = P(q), \]

where \( P \) is some yet unknown precorrection function. We assume \( P \) to be representable by a linear combination of basis functions \( P_n(q) \) as

\[ P(q) = \sum_n c_n P_n(q) = c \cdot P(q). \]

In our examples we later use the monomials \( P_n(q) = q^n \) as basis functions such that \( P(q) = c_0 + c_1 q + c_2 q^2 + \cdots \). Our theoretical derivation whose purpose it is to determine the coefficients \( c_n \) does not make any assumptions on the functional form of \( P_n(q) \), however. In the following, we want to correct for the cupping (or capping) in a 2D image \( f(r) \). The theory below can be applied to 3D volumes as well.

Making use of the linearity of the Radon transform \( R \) we define a set \( N+1 \) of basis images as

\[ f_n(r) = R^{-1} P_n(q), \]

which means that the images \( f_n \) are obtained by passing the raw data through the basis functions \( P_n \) followed by reconstruction. In total \( N+1 \) image reconstructions must be performed to obtain the basis images. In our case of using monomials, \( f_1 \) is the identity transform and hence \( f_1 \) corresponds to a reconstructed image without precorrection.

We want to find the set of coefficients \( c_n \) that minimizes the deviation between the linear combination

\[ f(r) = R^{-1} p = R^{-1} P(q) = \sum_{n=0}^N c_n f_n(r) = c \cdot f(r) \]

of the basis images and a given template image \( t(r) \). The template should represent the ideal image, without cupping, that we would like to obtain. The unknowns \( c \) can be determined by solving

\[ E^2 = \int d^2 r w(r)[f(r) - t(r)]^2 \leq \min, \]

where \( w(r) \) is a weight image which can be used to accentuate certain image areas. To find the minimum of \( E^2 \) we compute its gradient with respect to the vector of unknowns \( c \) and set it to zero:

\[ \nabla_c \int d^2 r w(r)[c \cdot f(r) - t(r)]^2 = 0. \]

This results in the linear system \( a = B \cdot c \) with

\[ a_i = \int d^2 r w(r)f_i(r)t(r), \quad B_{ij} = \int d^2 r w(r)f_i(r)f_j(r). \]

The solution to the optimization problem is simply given as

\[ c = B^{-1} \cdot a \]

and these coefficients are ready to be used for precorrection.

A potential drawback of ECC is the fact that it cannot perform channel-dependent correction. The explicit dependency of \( P(q) \) on the line of integration \( L \) is, however, sometimes required. Some reasons why the x-ray spectrum varies across the detector are the heel effect (anode angle effect), the use of bow-tie filters, varying detector efficiency as a function of the intersection length of the x-ray and the scintillator material and, last but not least, x-ray scatter. Tube current or even tube voltage modulation as a function of the rotation angle may be an issue, too. All these effects actually require a family \( P(L, q) \) of precorrection functions and not just one function \( P(q) \). Due to measurement errors and quantum noise an empirical determination of a channel-dependent precorrection function is much more difficult and far less elegant than determining a global \( P(q) \) as ECC does. Analytically one can derive \( P(L, q) \) given some assumptions about the anode material, anode angle, prefilteration, shaped filtration, detector absorption, and, potentially, scatter. As discussed initially, one does not exactly know these parameters and image quality is likely to remain imperfect. A hybrid cupping correction (HCC) method that combines such an analytical approach with ECC by applying them in some order to the raw data, say analytical precorrection followed by empirical precorrection \( P(P(L, q)) \), will help in these cases.9 For our TomoScope micro-CT scanner it turns out that ECC, which is the focus of this publication, is sufficient and for our particular scanner we found no significant benefit from using the proposed hybrid technique.

### III. WATER PRECORRECTION

A typical example of usage is the so-called water precorrection, the aim of which is to calibrate CT scanners such that no cupping artifacts in water or water equivalent materials are present anymore. To precorrect for water we use a calibration phantom that is a hollow cylinder filled with water. A measurement of that phantom yields the polychromatic projection values \( q \). To find the precorrection coefficients \( c \), we must also define the template \( t \) and the weight function \( w \). As previously mentioned, the basis functions \( P_n(q) = q^n \) are used.

The template \( t \) is defined to be a binary image with the density \( \mu_w \) inside the water phantom and zero outside. It is derived by a simple threshold-based segmentation of the reconstructed calibration phantom image \( f_j \) (which is a standard reconstruction). Unlike other empirical precorrection approaches, our method does thus neither rely on a specific phantom size or shape, or on special phantom positioning, nor does it require the exact knowledge of these parameters. The only requirement is that the object that serves as the calibration phantom must be homogeneous in some segmentable region and it must be of the same type of material one wants to precorrect for.

The weight function \( w \) shall account for the finite phantom thickness and for the smooth phantom edges due to the limited spatial resolution (point spread function effects). In those regions that neither belong to air nor to water, e.g., in the walls of the cylinder that may be made of polyethylene, \( w(r) = 0 \) is set. Outside the field of measurement \( w(r) = 1 \) as well. In the regions for which we are sure they definitely belong to water or air, we set \( w(r) = 1 \).
Given the template function \( t \) and weight function \( w \) plus the set of basis images \( f_n \), the coefficients can be readily determined as described in the previous section.

**IV. PRACTICAL CONSIDERATIONS**

We have tested the proposed precorrection algorithm using an in vivo small animal imaging micro-CT scanner (TomoScope 30s, VAMP GmbH, Erlangen, Germany) that is shown in Fig. 1. Our micro-CT is a dedicated high-speed in vivo mouse scanner equipped with a 1024 \( \times \) 1024 flat panel detector. The scanner is optimized for high dose efficiency and good low-contrast resolution and can perform complete scans in 20 s. The objects are placed on the object table which will be visible in the reconstructed images. The implementation of the precorrection procedure therefore has to take the object table into account. In principle, the weight can be set to \( w(r)=0 \) in the region of the table. However, one can do better when using the information given by the table pixels for calibration as well.

In the following we define the attenuation coefficient of water as \( \mu_W \). We further suppose the table to be homogeneous and made of a water-equivalent material with unknown attenuation coefficient \( \mu_T \). Thus, we can define the following template:

\[
t(r) = \begin{cases} \\
\mu_W & \text{for } r \in \text{ water phantom}, \\
\mu_T & \text{for } r \in \text{ table}, \\
0 & \text{for } r \in \text{ air},
\end{cases}
\]

for the minimization routine. The three regions are determined from a segmentation of the reconstructed image \( f_1 \).

To define the weight \( w(r) \), we erode the segmented water phantom by a circular structuring element of diameter \( d \) to hinder the phantom wall from contributing to our estimation. To remove the influence of spatial resolution, we further erode the three regions water, table, and air by a circular structuring element of diameter \( \Delta \), where \( \Delta \) exceeds the the full width of the system’s transversal point spread function by 10%. Moreover, we set \( w(r) \) to zero in regions outside the field of measurement (FOM). Thus we have

\[
w(r) = \begin{cases} \\
1 & \text{for } r \in \text{ water, table, or air after erosion}, \\
0 & \text{for } r \in \text{ eroded boundaries and outside the FOM}.
\end{cases}
\]

An estimation procedure to find the table attenuation coefficient \( \mu_T \) remains to be made. The desired value can be obtained by simply minimizing for \( N+2 \) parameters instead of minimizing for \( N+1 \). We identify the last coefficient with the unknown table attenuation coefficient value: \( c_{N+1}=\mu_T \). Since the template can be decomposed as \( t(r)=t'(r)+\mu_T t''(r) \), we set a new reconstructed image \( f_{N+1}(r)=-r''(r) \).

Then we can solve

\[
\int d^2 r w(r) [c \cdot f(r) - t'(r)]^2 = \min
\]

as a \((N+2)\)-dimensional problem in complete analogy to the \((N+1)\)-dimensional case we originally started with.

**V. RESULTS**

The precorrection method was tested using data of the TomoScope 30s micro-CT scanner. This scanner has a 2D detector and therefore we have a cone-beam geometry allowing for 3D volume acquisition. A water phantom with \( D=32 \) mm outer diameter and \( d=0.5 \) nm wall thickness designed for homogeneity and noise assessment in micro-CT (QRM GmbH, Möhrendorf, Germany) was scanned at a tube voltage of 40 kV (figure 1). The polynomial degree was set to \( N=4 \) which has proven to provide good results. Figure 2 illustrates the intermediate steps of the empirical cupping correction.

Care has to be taken when the noise level in the images \( f_n(r) \) is too high to allow for an accurate correction. One can
avoid this by either averaging more than one calibration scan, by averaging over adjacent slices under the assumption that the phantom is invariant under translations along the longitudinal axis, or by using a smoother reconstruction kernel. In our case we use a smooth reconstruction kernel and then average over 50 adjacent reconstructed slices to obtain low-noise versions of \( f_{\text{r}} \).

Using these averaged basis images, the following calibration parameters were obtained

\[
\mathbf{c} = \begin{pmatrix}
-0.00247761 \\
0.399786 \\
-0.0661509 \\
0.121149 \\
-0.0295839
\end{pmatrix}
\quad \text{and} \quad
\mu_t = 1.175 \mu_W.
\]

They are valid for our specific scanner and scan protocol and may serve as a typical example. The maximum polychromatic attenuation for this calibration phantom was \( q_{\text{max}} = 1.8 \). This value can be understood by the following rough estimate. At 40 kV our scanner has an effective energy of 25 keV in the center of the water phantom, which means the effective attenuation of water is about 0.05/mm. The ray that suffers from the maximum attenuation runs through 32 mm of water and two times the 3 mm table.

Figure 3 shows the corrected image \( f(\mathbf{r}) = \mathbf{c} \cdot f(\mathbf{r}) \). No cupping is visible, water is at 0 HU, and the table at 175 HU. In Fig. 4 the profiles through the central column of the images with and without precorrection, i.e., through \( f_1(\mathbf{r}) \) and \( f(\mathbf{r}) \), with these parameters confirm the results.

To demonstrate the performance using objects different from the calibration phantom we scanned a 20 mm water phantom. Figure 5 shows the results of a 32 mm versus a 20 mm water phantom with and without ECC. The calibration coefficients were determined from the 32 mm scan. The cupping is removed in both cases.

---

**Fig. 3.** The corrected image \( f(\mathbf{r}) = \mathbf{c} \cdot f(\mathbf{r}) \) apparently shows no cupping anymore. We used the same gray level window strategy as in Fig. 2, namely mean plus minus two standard deviations determined in the phantom area. Since no cupping is present the window width becomes rather small and the phantom looks quite noisy.

**Fig. 4.** Column profiles of the images \( f_1 \) (original image) and \( f \) (corrected image) of the water phantom of Fig. 2. The corrected image has been obtained from the coefficients obtained by averaging 50 slices. Note that the rightmost pixels correspond to the table that is below the phantom. In the corrected image the table’s mean value is 175 HU whereas the water is 0 HU.

**Fig. 5.** Water phantoms of 32 and 20 mm outer diameter with and without cupping correction. The coefficients \( \mathbf{c} \) were determined using the 32 mm data. Cupping is apparent but less pronounced for the smaller phantom. ECC completely removes the cupping in both cases. The uncalibrated images were scaled to lie close to 0 HU in the water area. The gray scale window center was set to 0 HU and the window width to 500 HU in all four images.
An example of a mouse scan acquired on this micro-CT scanner at 40 kV is shown in Fig. 6. For this scan, the maximal attenuation value was $q=2.4$, which is larger than the maximal attenuation value $q_{\text{max}}$ taken for calibration. Outside the calibrated range $[0; q_{\text{max}}]$ we replace the correction polynomial by a linear extrapolation. The top images show the reconstruction when no precorrection is performed and the bottom row shows the precorrected data. The cupping artifacts are less apparent in such inhomogeneous objects, but a close look reveals that they are removed. A more obvious indicator showing that the ECC method is a first-order compensation for the polychromatic effects of the spectrum are the beam-hardening artifacts that appear as dark streaks between the bones. These are greatly reduced, although a complete removal is possible only with higher order and therefore iterative beam-hardening correction approaches.

VI. CONCLUSION AND DISCUSSION

The empirical cupping correction is an efficient raw-data-based CT calibration method. In contrast to other methods, ECC makes no assumptions on the size and positioning of the calibration phantom and it does not need to know the x-ray spectrum. The method requires nothing but a set of reconstructed images and it is quite simple since it solves only a linear system. Once the coefficients $c$ are determined, precorrection means passing the acquired data $q$ through the linear combination $P(q) = c \cdot P(q)$ of basis functions, which is computationally highly efficient.

It was demonstrated that cupping can be almost completely removed. Since our algorithm is an empirical correction it does not only correct beam-hardening-based cupping but also cupping due to scattered radiation or cupping due to normalization errors. ECC cannot distinguish between these sources of error. In the case of scatter it may be of advantage to choose a calibration phantom whose cross section and dimension are of representative size to mimic the targeted imaging situation as closely as possible. Another important aspect is the restoration of the true CT values. Only with properly precorrected data can quantitative data analysis be performed. If a channel-dependent precorrection is required, ECC can be combined with analytical precorrection methods to obtain a hybrid technique that comprises the advantages of both approaches.\(^9\)

Summarizing, the empirical cupping correction approach provides an easy way to guarantee cupping-free and beam-hardening-reduced images with highly accurate and well-calibrated Hounsfield values.

---

\(^9\) Electronic mail: marc.kachelriess@imp.uni-erlangen.de


