I. INTRODUCTION

Coronary artery disease is one of the most important causes of death in western civilizations. Therefore, noninvasive techniques for imaging the heart and for early diagnosis of disease are still in high demand. In addition to magnetic resonance and ultrasound imaging, x-ray based techniques remain important as they exclusively allow to image and, resonance and ultrasound imaging, x-ray based techniques disease are still in high demand. Therefore, noninvasive Technics for imaging the heart and for early diagnosis of death in western civilizations. Therefore, noninvasive noninvasive coronary artery disease is one of the most important causes of death in western civilizations. Therefore, noninvasive techniques for imaging the heart and for early diagnosis of disease are still in high demand. In addition to magnetic resonance and ultrasound imaging, x-ray based techniques remain important as they exclusively allow to image and, potentially, to quantify coronary calcification.

Fluoroscopy, conventional computed tomography (CT), spiral CT, and electron-beam computed tomography (EBT) are the corresponding imaging techniques available today. They are not fully established yet due to the following reasons: either the method fails to detect small amounts of calcium (fluoroscopy), or the systems are hardly available (EBT) due to high cost, or it is not possible to scan the whole region of interest in adequate quality during a single breath-hold (conventional CT).

The development of new continuously rotating scanners and the introduction of spiral CT1,2 led to a renaissance of computed tomography. The volume scanning capability offered by spiral CT in combination with an increase in rotation speed (subsecond scanning is available on many scanners today) have made volume scans of whole anatomical regions during a single breath-hold a clinical reality.

Nevertheless the use of standard spiral reconstruction techniques in heart imaging does not lead to satisfactory results, because scan times, although typically less than or equal to 1 s per 360° rotation, are still of the order of one heart cycle. Even for 180° algorithms a data range acquired over 2×(180°+Φ)/360°×trot is accessed where Φ is the fan angle and trot is the time for a 360° rotation. Thus for a 0.75 s scanner standard spiral reconstruction will use data covering slightly less than 1 s. Motion and inconsistency artifacts are necessarily introduced.

Temporal resolution of <100 ms would be desirable to image the heart with high fidelity, but conventional CT scanners will not offer such high speed in the foreseeable future. Therefore we propose a new approach: partial scan reconstruction or modified z-interpolation with electrocardiogram (ECG) gated selection of corresponding heart phases. We assumed that dedicated algorithms 180°CD (Cardio Delta, a partial scan reconstruction) and 180°CI (Cardio Interpolation, a piecewise linear interpolation between successive spiral data segments belonging to the same heart phase) improve the imaging quality of the heart significantly.

In this paper we will present specifications of the corresponding algorithms and the resulting slice sensitivity profiles (SSP), as well as processed patient data to compare these new approaches to the standard spiral algorithm 180°LI (Linear Interpolation).

II. MATERIALS AND METHODS

Notations and definitions used throughout this paper are given below. We will express some quantities by the corresponding rotation angle (e.g., slice thickness will not only
appear as \( S \), but equivalently as \( \Delta \alpha_2 = 2\pi S/d \) and switch, if convenient, from one to the other representation.

\[ n \]
convolution symbol

\[ \ast \]
\( n \)-fold self-convolution of function 

\[ \delta(\cdot) \]
Dirac's delta function

\[ \Pi(\cdot) \]
rectangle function of height 1 and support \([-1,1]\).

\[ \Lambda(\cdot) \]
triangle function of height 1 and support \([-1,1]\).

\[ \Pi_C \]
Dirac's delta function

\[ \Pi_C(\cdot) \]
Dirac's delta function

\[ \Pi_C(\cdot) \]
rectangle function of height 1 and support \([-1,1]\).

\[ \Lambda(\cdot) \]
triangle function of height 1 and support \([-1,1]\).

\[ \lfloor \cdot \rfloor \]
floor function, yields greatest integer lower or equal.

\[ 
- \]
ceil function, yields smallest integer greater or equal.

\[ x \lor y \]
maximum of \( x \) and \( y \), \( x \lor y = \max(x,y) \)

\[ x \land y \]
minimum of \( x \) and \( y \), \( x \land y = \min(x,y) \)

\[ \text{iff} \]
if and only if

\[ t_{rot} \]
time for a 360° rotation

\[ d \]
table increment per 360° rotation

\[ u \]
table speed, \( u = \frac{d}{t_{rot}} \)

\[ u_{\min} \]
minimal table speed necessary to cover the heart during a single breath-hold

\[ S \]
nominal slice thickness

\[ p \]
pitch, \( p = \frac{d}{S} \)

\[ w',w'' \]
z-interpolation weights, \( w' + w'' = 1 \)

\[ z \]
axis of rotation

\[ z \]
position of detector array center,

\[ z = (\alpha/\pi d) \]

\[ z_R \]
arbitrarily selectable reconstruction position,

\[ z_R = \frac{(\alpha_R/\pi d)}{} \]

\[ \Phi \]
fan angle, in our case \( \Phi = 52° \)

\[ \beta \]
angle within fan, \( \beta \in [-\frac{1}{2}\Phi,\frac{1}{2}\Phi] \)

\[ \alpha \]
projection angle, \( \alpha \in [0,2\pi] \) for a 360° scan, \( \alpha \in \mathbb{R} \) for a spiral scan

\[ \alpha_R \]
angle associated with reconstruction position

\[ \Delta \alpha_S \]
angle of rotation while advancing the table by \( S \), \( \Delta \alpha_S = \frac{2\pi S/d}{2\pi p} \)

\[ \alpha_k,\beta_k \]
angles suitable for interpolation at \( \alpha,\beta \) with \( k \in \mathbb{Z} \)

\[ \text{SSP}(\cdot) \]
slice sensitivity profile, either as a function of position \( z \) or angle \( \alpha \)

\[ P(\beta,\alpha) \]
spiral projection data

\[ P_X(\beta,\alpha,z_R) \]
interpolated raw data at position \( z = z_R \) for algorithm \( X \)

\[ f_H \]
patients heart rate (may vary during the scan), typically \( 60 \text{ min}^{-1} \leq f_H \leq 100 \text{ min}^{-1} \)

\[ \Delta \alpha_H \]
angle of rotation during one heart beat, \( \Delta \alpha_H = \frac{2\pi f_H t_{rot}}{2\pi} \)

\[ o,w \]
relative offset and width of the selected interval in the cardiac R-R cycle, \( o \in [0,1) \), \( w \in (0,1) \)

\[ L_{o,w} \]
allowed data range

\[ n_{\text{max}} \]
maximal number of rotations on both sides of \( z_R \) to be taken into account for interpolation

\[ d_{\text{max}} \]
maximal distance from the reconstruction plane for data points to be taken for interpolation, \( d_{\text{max}} = n_{\text{max}} d \)

\[ \Delta \alpha_{\text{max}} \]
to restrict the interpolation at position \( \alpha_R \) to the range \([\alpha_R - \Delta \alpha_{\text{max}},\alpha_R + \Delta \alpha_{\text{max}}]\).

\[ \Delta \alpha_{\text{max}} = 2m_{\text{max}} \pi + \Phi \]

\[ w_{\text{slic}} \]
lower limit for \( w \) to assure a certain overlap between the slice and the selected data range

\[ w_{\text{opt}} \]
lower limit for \( w \) for optimal data filling

\[ w_{\text{res}} \]
lower limit for \( w \) to restrict the data range to \( n_{\text{max}} \) rotations on either side of \( z_R \)

\[ w_{\text{triv}} \]
trivial upper limit for \( w \), for \( w \gg w_{\text{triv}} \) the trivial case occurs: only one heart cycle contributes

All scans were performed on a subsecond spiral CT scanner (SOMATOM PLUS 4, Siemens Medical Systems, Erlangen, Germany) using \( d = 3 \text{ mm} \) table increment per 360° rotation, nominal slice thickness \( S = 3 \text{ mm} \), i.e., pitch \( p = d/S = 1 \), rotation time \( t_{rot} = 0.75 \text{ s} \). The slice thickness was selected as \( 3 \text{ mm} \) in order to cover the complete heart (typically 12 cm to 15 cm) during a single breath-hold while keeping the pitch as low as possible. A PC station was used to simultaneously record the electrocardiogram (ECG) and the x-ray-on signal delivered by the data acquisition system for synchronization purposes. We used our own software to automatically detect the R-peaks from the ECG data (these are the most pronounced peaks of the ECG and thus easier to detect than other ECG peaks) and to perform synchronization with the projection numbers.

Two freely selectable parameters are used to select the cardiac phase: the offset \( o \) and width \( w \). The width determines the fraction \( (0 < w < 1) \) of one cardiac cycle to be used, the offset determines the relative position \( (0 \leq o < 1) \) within the R-R cycle as is depicted in the lower part of

![Fig. 1. Algorithm 180°CI. Only data belonging to user-selected ECG intervals are allowed for reconstruction. The allowed data ranges are determined by choosing offset \( o \) (here: \( o \approx 18\% \)) and width \( w \) (here: \( w \approx 68\% \)) with respect to the R-R cycle. Three interpolation examples are shown: \( P_1 \) and \( P_2 \): interpolation cannot be done between directly neighboring measured and virtual spiral segments. The algorithm has to access \( P_1 \) and \( P_2 \), respectively. Therefore the interpolation length is 360°. \( P_3 \): direct interpolation as in 180°LI. The rebinned spiral is shown only for central rays, i.e., for \( \beta = 0 \).](image-url)
Fig. 1. Of course we try to select primarily a phase with relatively little motion. For example, \( o = 20\% \) and \( w = 70\% \) means that the range from 20\% to 90\% (i.e., 70\% of total cycle) is accepted and the first 20\% and the last 10\% of each \( R-R \) cycle are excluded. It has to be emphasized that the ECG information is used only to synchronize the algorithm. Parameters \( o \) and \( w \) are used to navigate windows of acceptable projections relative to the ECG. We define the set \( L_{o,w} \) to be the set of all projection angles \( \alpha \) that are to be used for reconstruction:

\[
L_{o,w} = \{ \alpha | \text{projection} \, \alpha \, \text{lies in one of the allowed } o-w \text{-windows} \}.
\]

We assume \( L_{o,w} \) to be a union of disjunct intervals of finite measure.

### A. Reconstruction algorithms

In order to synthesize planar data at position \( (\beta, \alpha, z_R) \) we can do an interpolation between neighboring data points obtained at the same angle of rotation \( \alpha + 2\pi z \) (360° algorithms) and points rebinned from opposite views \( \alpha + 2\beta + \pi + 2\pi z \) (180° algorithms) as described in Ref. 3. Thus data points at the following positions \( (\beta_k, \alpha_k) \) are available for interpolation:

\[
\beta_k = (-1)^k \beta,
\]

\[
\alpha_k = \alpha + 2 \delta_{\text{odd}, k} \beta + k \pi,
\]

where \( k \in \mathbb{Z} \). We define the indicator function

\[
\delta_{\text{odd}, k} = \begin{cases} 1 & \text{if } k \in 2\mathbb{Z} + 1 \\ 0 & \text{elsewhere} \end{cases},
\]

which indicates whether the measured spiral or the rebinned data set from opposite views is used. Since the notation is very compact, we want to emphasize once more that the definitions of the \( \beta_k \) and \( \alpha_k \) include data rebinned from opposite views. The fact that all algorithms make use of rebinned data is not explicitly stated below.

#### 1. Dedicated cardiac interpolation algorithm 180°CI

180°CI is a modification of the most commonly used \( z \)-interpolation algorithm 180°LI: If an interpolation point falls into a forbidden region (i.e., \( \alpha \notin L_{o,w} \)) it is skipped and the next allowed one is chosen (see Fig. 1). The search has been limited to a maximum range which can be selected by the user. In the cases presented below we chose a maximum distance of \( d_{\text{max}} = 6 \text{mm} \) from the reconstruction plane, corresponding to \( \Delta \alpha_{\text{max}} = 4\pi \) and \( n_{\text{max}} = 2 \).

Mathematically the interpolation at the reconstruction position \( \alpha_R \) can be written as

\[
P_{\text{CI}}(\beta, \alpha, \alpha_R) = w' P(\beta_{k'}, \alpha_{k'}) + w'' P(\beta_{k''}, \alpha_{k''}),
\]

where \( k' \) and \( k'' \) depend not only on \( \alpha_R \) but also on the indicator set \( L_{o,w} \):

\[
k' = \sup \{ k | \alpha_k < \alpha_R, \, \alpha_k \in L_{o,w} \},
\]

\[
k'' = \inf \{ k | \alpha_k > \alpha_R, \, \alpha_k \in L_{o,w} \}.
\]

The weights are calculated as usual:

\[
w' = 1 - w'' = \frac{\alpha_k - \alpha_R}{\alpha_k - \alpha_{k'\text{-w}}},
\]

(1c)

The synthesized projections \( P_{\text{CI}} \) then undergo the standard image reconstruction process, e.g., filtered backprojection. The reconstruction position \( z_R \) can be chosen arbitrarily and retrospectively as customary in spiral CT. The effective temporal resolution that can be achieved by 180°CI is given by the fraction \( w \) of the cardiac cycle that is used for reconstruction and thus results as \( w/f_H \).

Since \( L_{o,w} \cap R \) for \( w \uparrow 1 \), the cardio interpolation algorithm 180°CI approaches the 180°LI standard linear interpolation algorithm when the width of the allowed data ranges approaches 100\%.

As shown in Appendix A, algorithm 180°CI can also be implemented as a spiral completion algorithm. This means that exactly those parts of the spiral which do not lie in an allowed data range are synthesized by interpolation between neighboring data in allowed ranges to obtain a complete spiral data set. Applying a 180°LI \( z \)-interpolation to this precorrected data is equivalent to the described algorithm 180°CI: however, any other spiral interpolation algorithm can be used just the same. This method has implementational advantages: only 100(1 - \( w \))\% of the data have to be interpolated (which is in our case done on a standard workstation.) Then the data can be put back to a reconstruction unit to undergo \( z \)-interpolation and reconstruction. This means that pipelining the process is possible.

### 2. Prediction of slice sensitivity profiles for 180°CI

Let SSP(\( z \)) denote the slice sensitivity profile as a function of the table position \( z \). It is defined to be the reconstruction of a delta object at the origin of the coordinate system. Therefore the corresponding projection is

\[
P(\beta, \alpha) = \delta (\beta) \delta (\alpha) \star \Pi \left( \frac{\alpha}{\Delta \alpha_S} \right) = \delta (\beta) \Pi \left( \frac{\alpha}{\Delta \alpha_S} \right).
\]

(2)

We here assume for the sake of simplicity that the original slice sensitivity profile, i.e., the SSP in conventional CT without table motion, is represented adequately by the rectangle function \( \Pi (\cdot) \).

The calculation of the SSP for the 180°CI algorithm is not straightforward because the parameters \( o, w \) and the patient's heart rate \( f_H \) play a significant role. Nevertheless the calculation can be carried out explicitly and the result has the form

\[
\text{SSP}(\alpha) = \sum_{i \in L} \Lambda \left( \frac{\alpha}{m_i \pi} \right) \star \Pi \left( \frac{\alpha - c_i}{2 d_i} \right)
\]

(3)

with coefficients \( c_i, d_i \in \mathbb{R} \) and \( m_i \in \mathbb{N} \). This can be made clear in the following way: for each \( \alpha, 180° \)CI has to find appropriate interpolation points that are contained in \( L_{o,w} \). These points are separated by multiples of 180°: \( m_i \pi \) [only central rays, i.e., \( \beta = 0 \), contribute to projection (2)]. Whenever \( m_i \) is even, this means that both points lie either on the measured or on the rebinned spiral. If \( m_i \) is odd, one point lies on the measured and the other one on the calculated spiral. Furthermore \( m_i \) will be constant for a certain range of
α which we denote by \([c_i - d_i, c_i + d_i]\). Each one of these ranges gives a contribution \(\Lambda(\alpha/m, \pi) = \Pi((\alpha - c_i)/2d_i)\) to the slice sensitivity profile. The SSP then consists of a sum of such contributions with different parameters \(c_i, d_i\) and \(m_i\) which results in Eq. (3).

Having no simple expression of how to determine the \(m_i\) is the main problem when calculating the SSP for arbitrary parameters \(α, w,\) and \(f_H\). Convolution of Eq. (3) results in a \(22\) function which is given in detail in Appendix B, Eq. (B1). A sum over such functions, as appears in (3), would be very uninteresting, however, and therefore we will restrict our following considerations to one special, but very instructive case.

Let us assume that the heart rate \(f_H\) remains constant during the scan and that it satisfies the condition \(f_H\text{rot}=1\). The angular range \(Δα_H\) covered during one heartbeat is then \(Δα_H=2\pi\). We will now consider the case where only half of the data is used for reconstruction of the delta object:

\[
l_{o,1/2} = \cup \{2\pi(o+i), 2\pi(o+i) + \pi\}. \tag{4}
\]

Therefrom we gain an enormous simplification in calculating the SSP: the denominator \(\alpha_{o,1/2} - \alpha_{o,1/2}\) of Eq. (1c) has the value \(2\pi\) for all reconstruction positions \(\alpha_{o,1/2}\). In other words: \(m_i = 2\\\\forall i\) in Eq. (3).

In Eq. (4) we have assumed without loss of generality one-

The projection data (2) has the support \([-Δα_S/2, Δα_S/2]\) in \(α\). Applying 180°CI with the above mentioned settings is now equivalent to truncating the support to

\[
[-Δα_S/2, Δα_S/2] \cap l_{o,1/2},
\]

and convolving with a triangle function of width \(2\times2\pi\). The truncated projection data consist of a sum of rectangle functions

\[
P_{CI}(β, α) = δ(β)\Pi(\alpha/Δα_S) \sum_{i = i_{\text{min}}}^{i_{\text{max}}} \Pi((α-c_i)/2d_i)\]

\[
= δ(β) \sum_{i = i_{\text{min}}}^{i_{\text{max}}} \Pi((α-c_i)/2d_i),
\]

with \(i_{\text{min}} = [-\frac{1}{2}(p^{-1} + 1 + 2o)]\) and \(i_{\text{max}} = [\frac{1}{2}(p^{-1} - 2o)]\). The rectangle functions have the support

\[
\pi[max(-p^{-1}, 2o + 2i), min(p^{-1}, 2o + 2i + 1)];
\]

hence

\[
c_i = \frac{1}{2}\pi(min(p^{-1}, 2o + 2i + 1) + max(-p^{-1}, 2o + 2i))\]

\[
d_i = \frac{1}{2}\pi(min(p^{-1}, 2o + 2i + 1) - max(-p^{-1}, 2o + 2i)).
\]

The slice sensitivity profile can now easily be calculated by convolving with a triangle function of width \(2\times2\pi:\)

\[
\text{SSP}(α) = \sum_{i = i_{\text{min}}}^{i_{\text{max}}} \Pi(α/c_i)\Lambda(α/2π)\].
\]

Using the identity \(\Lambda(α/2π) = (1/2π)\Pi(α/2π)\) makes it possible to use Eq. (B1) from Appendix B and gain the expression

\[
\text{SSP}(α) = \frac{1}{2\pi} \sum_{i = i_{\text{min}}}^{i_{\text{max}}} \Pi(2\pi, 2\pi, 2d_i(α - c_i)).
\]

The coefficients \(c_i\) and \(d_i\) and consequently SSP depend on the offset \(o\) and on pitch \(p\).

Figure 2 shows contour plots of the slice sensitivity profiles (normalized to the overall maximum) as a function of \(z\) and offset \(o\) for pitch \(p = \frac{3}{2}\), \(p = 1\), and \(p = \frac{1}{2}\). A 3D plot is shown as well (for \(p = 1\)), since 3D plots can be associated better with the two-dimensional SSP plots most of us are used to. There are two ways to interpret such plots. First, the ordinate can be regarded as being the offset \(o\) which modulates the overlap of the allowed intervals with the slice containing the delta object. This is the way we have derived the SSP. The other point of view is to say: this is the SSP graph for a given offset \((o\text{ fixed})\), but for varying positions of the delta peak. Then the ordinate has the meaning of being the position of the delta object (in units of \(Δα_H\)) with respect to the cardiac phases. This is why we have decided to leave away the label for the ordinate and why we were free to center the plots vertically.

For more clarity let us now interpret Fig. 2(a) as an example: The area between the two vertical lines, which we have additionally inserted, represents the slice containing the delta peak. In the range from 0% to 25% and from 75% to 100% (representing either offset \(o\) or distance of the delta peak as we have discussed in the previous paragraph) an allowed interval lies completely within the slice (see the inclined lines which represent \(l_{o,1/2}\)). The location of the SSP’s maximum follows these intervals; in other words; it moves to the right with increasing offset. When going from 25% to 75% a transition takes place, since another allowed interval intersects the slice at its left edge. The maxima thus start to move toward this edge, additionally we find a decrease of the maximum value of about three contours (\(\sim 30\%\)) with respect to the overall maximum when approaching the 50% ordinate.

When we have a look at the plot for \(p = \frac{3}{2}\) [Fig. 2(d)] we find a more pronounced decrease of the maximum value whereas the maximum is almost constant for small pitch values such as \(p = \frac{1}{2}\) [Fig. 2(c)]. The variation of the maximum value obviously depends on the overlap of the slice and the allowed ranges. Of course, cases with zero overlap would be
fatal: calcifications would not be visible. The slice overlap condition, introduced in the following section, helps to avoid this case.

The FWHM as a function of $o$ is given in Fig. 3. We find: (a) constant FWHM for $p = \frac{1}{2}$ and no difference of the FWHM value compared to LI; (b) the $p = 1$ values vary with $o$ and are slightly increased in comparison to the standard $z$-interpolation; and (c) variations with the offset and 50% average increase as compared to 180°LI for $p = \frac{3}{2}$.

To demonstrate some slice sensitivity profiles for the general case of arbitrary heart rate and width, we have included the contour plots of the corresponding simulated SSPs for

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**Fig. 2.** Behavior of SSP($z$) as a function of offset $o$ and position $z$ for the special case $f_{rot}=1$ and $w=\frac{1}{2}$. SSPs are normalized to $[0,1]$. (a) Contour plot of SSP($z$) as a function of $o$ and $z$ for $p = 1$. Contours are shown at 5%, 15%, 25%, etc. of the SSP’s maximum value. The inclined lines depict the borders of $L_{o,1/2}$ and therefore move to the right with increasing offset $o$. In this plot we have additionally added two vertical lines at $z = \pm \frac{1}{2}S$ to show the slice containing the delta peak. (b) 3D plot of the same profiles. The deformation of SSP when the offset approaches the worst case, where the delta peak lies in the center of a forbidden interval (at the 50% ordinate), can be seen clearly. (c) Plot for $p = \frac{1}{2}$ and (d) for $p = \frac{3}{2}$.
each patient presented in Sec. III. It will be seen that the profiles we have derived in this section (under the restrictions $f_{H \text{rot}} = 1$ and $w = 1$) qualitatively describe the common case very well.

3. Parameter settings for 180°CI

In the following paragraphs several conclusions for the general case (arbitrary values for width $w$, offset $o$, pitch $p$ and heart rate $f_H$) are drawn. The result is the system of Eq. (6) that describes the dependencies between the scan parameters and reconstruction parameters. Further on we will discuss the solutions of (6) assuming two cases: (a) that the scanner parameters (such as $S$, $t_{rot}$, and $p$) can be varied continuously and (b) that there are only a few discrete settings available.

a. Slice overlap. We must be sure that the maximum SSP value is always greater than zero, or in other words, that details, e.g., lesions, calcifications, etc., are not hidden completely in forbidden data ranges. This can be achieved by choosing $w$ so that the gaps of $L_{o,w}$ do not exceed slice thickness $\Delta \alpha_S$. We simply demand $(1-w)\Delta \alpha_H < \Delta \alpha_S$.

Further on, the overlap of $L_{o,w}$ with the slice should not only be greater than zero; rather it should be as large as possible. The larger the overlap is, the smaller the variation (with respect to $o$) of the SSP’s maximum value will be. We then can replace $(1-w)\Delta \alpha_H < \Delta \alpha_S$ by the stronger condition $(1-w)\Delta \alpha_H < \Delta \alpha_S$ which yields, when solved for $w$,

$$w \gg w_{\text{slice}} = 1 - \frac{\Delta \alpha_S}{\Delta \alpha_H} = 1 - \frac{f_{H \text{rot}}}{p}.$$ 

In our case, this was not really a restriction; $w \gg 0.25$ suffices for the typical patient ($f_H \gg 60 \text{ min}^{-1}$, $p = 1$, $t_{rot} = 0.75$ s).

b. Optimal data filling. The optimal situation would occur whenever successive intervals of $L_{o,w}$ are separated by a distance of $2\pi$: Data still missing after one rotation would be available at the right position during the next rotation. (We here neglect the situation, where acceptable data are available already after a 180° rotation. This would demand heart rates of the order of half a rotation time, i.e., in our case $f_H \approx 160 \text{ min}^{-1}$. Cases which need more than one rotation are neglected as well because they correspond to patients with very low heart rates and can be met by increasing $t_{rot}$.) Therefore $2\pi \pm w \Delta \alpha_H = \Delta \alpha_H$ is necessary, which means

$$w = w_{\text{opt}} = \left| 1 - \frac{2\pi}{\Delta \alpha_H} \right| = \left| 1 - f_{H \text{rot}} \right|.$$ 

A freely adjustable $t_{rot}$ would lead to a free choice of $w$; less than 20% of the heart cycle would be desirable, i.e., $w < 0.2$.

c. Range restriction and trivial case. Let us now calculate the data range necessary on either side of $z_R$ to have a complete set of data assuming that $w \gg w_{\text{opt}}$. Furthermore, we consider only cases with $w \Delta \alpha_H = \pi + \Phi$, i.e., the trivial case where less than one heart cycle is needed to collect a complete data set is excluded:

$$w \leq w_{\text{triv}} = \frac{\pi + \Phi}{\Delta \alpha_H}.$$ 

During the first heart cycle $w \Delta \alpha_H$ data are acquired. Each following cycle will contribute a data range of only $w_{\text{opt}} \Delta \alpha_H$, since the scanner has to advance by an angle of $2\pi$ before requiring new data. Exactly this is ensured by $w_{\text{opt}}$ (see above). Therefore, to acquire a range of length $\pi + \Phi$,

$$1 + \frac{\pi + \Phi}{w_{\text{opt}} \Delta \alpha_H} = \frac{\Delta \alpha_H + \frac{\pi + \Phi - w \Delta \alpha_H}{w_{\text{opt}} \Delta \alpha_H} \leq \Delta \alpha_{\text{min}}.}$$ 

Solving for $w$ yields

$$w \equiv w_{\text{res}} = w_{\text{opt}} + \frac{\pi + \Phi - \Delta \alpha_{\text{max}} w_{\text{opt}}}{\Delta \alpha_H}.$$ 

Obviously with an increasing number of rotations necessary the $z$-resolution decreases. We conclude: an improved time resolution ($w_{\text{opt}}$ as small as possible) will lead to a decreased $z$-resolution and vice versa. As we will see in Sec. III, artifacts are likely to be as significant as $z$-resolution.

To sum up, we have the following conditions imposed on $w$: 

\[ \text{Fig. 3. Plot of the FWHM as a function of } \rho \text{ for the special case } f_{H \text{rot}} = 1 \text{ and } w = \frac{1}{2}. \text{ The solid lines represent the FWHM for } p = \frac{1}{2}, \text{ } p = 1, \text{ and } p = \frac{3}{2}, \text{ respectively (from bottom to top). For comparison we have included the FWHM of the standard algorithm 180°LI for } p = \frac{1}{2} \text{ (dashed line). For } p = \frac{1}{2} \text{ and } p = 1 \text{ it is equal to } S \text{ and it thus coincides with the plot of 180°CI at } p = \frac{3}{2}. \]
These conditions restrict the range of allowed $w$ such that
enough acceptable data will be within the maximum distance
$\Delta \alpha_{\text{max}}$ from the reconstruction plane. The general case of
varying heart rate deviates only slightly from (6) since to our
experience the patients heart rate can be assumed to be lo-

cally constant (i.e., constant within $2n_{\text{max}}$ rotations). Of
course, our software handles the general case of arbitrarily
varying ECG frequency as well, by computing the selectable
range of widths $w$ as a function of the reconstruction position
$z_R$. Moreover, the user is even allowed to break the rule $w
\leq w_{\text{triv}}$, since increasing $w$ means smaller forbidden data
ranges and this may improve the image quality, although it
affects the temporal resolution negatively.

Equations (6) need further discussion, since they connect
the most important scan parameters: $p$, $S$, and $t_{\text{rot}}$. These are
the key parameters that determine heart coverage. High set-
tings for the pitch $p$ as well as low rotation times $t_{\text{rot}}$ are
desirable to scan a large volume within a given time (single
breath-hold) and desired $z$-resolution. For cardiac imaging a
high temporal resolution, i.e., a small time window of width
$w$ is necessary as well. These are conflicting demands, as we
have $w \gg 1 - f_{Ht_{\text{rot}}} / p$: a high pitch setting implies a high
rotation time (a slow rotation speed) and vice versa; thus
there is no gain in volume coverage. For those cases, where
the given heart rate $f_H$ and the available settings for $t_{\text{rot}}$
(which is usually not arbitrarily selectable on commercial CT
scanners) really mean a restriction according to (6), we sug-

gest to use a small value for the pitch and, in order to achieve
complete heart coverage, a higher slice thickness $S$ to keep the
table increment $d = pS$ constant.

To become quantitative we have depicted system (6) of
inequalities in Fig. 4. The shaded areas show the allowed
settings for the width $w$ as a function of $f_{Ht_{\text{rot}}}$. It is impor-
tant to choose $w$ as small as possible, since $w$ gives the
relative fraction of the heart cycle that will be used for re-
construction.

Assuming that $t_{\text{rot}}$ can be adjusted continuously, we can
reach the minimal achievable width (for a given $n_{\text{max}})$

$$w = w_{\text{min}} = \frac{\pi + \Phi}{\Delta \alpha_{\text{max}}},$$

by selecting the rotation time such that

$$f_{Ht_{\text{rot}}} = 1 \pm w_{\text{min}} = 1 \pm \frac{\pi + \Phi}{\Delta \alpha_{\text{max}}},$$

(cf. Fig. 4). To cover the complete heart (12 cm–15 cm)
during a single breath-hold time (30 s–40 s) a table speed $v$
of at least $v_{\text{min}} = 4 \text{ mm/s}$ is necessary and

$$pS = v_t_{\text{rot}} \geq v_{\text{min}} t_{\text{rot}}$$

has to hold. Up to here, we have derived expressions for the
table increment $d = pS$ and rotation time $t_{\text{rot}}$ to give an opti-
mal temporal resolution. The slice overlap condition $w
\gg w_{\text{slice}}$ now translates into

$$p < \frac{f_{Ht_{\text{rot}}}}{1 - w} = \frac{1 \pm w_{\text{min}}}{1 - w_{\text{min}}}.$$  

Equations (7) could be used to determine scan parameters
$p$, $S$, and $t_{\text{rot}}$ that result in optimal temporal resolution $w
= w_{\text{min}}$. Unfortunately, on commercially available scanners,
the parameters cannot be selected arbitrarily; rather the
user’s choice is restricted to a discrete subset of parameter
values. In general, it will not be possible to select a rotation
time, such that (7b) can be fulfilled. We have therefore
determined the “optimal” values assuming that

$$t_{\text{rot}} \in \{0.75, 1, 0.1, 0.25, 1.5 \} \text{ s},$$

$$S \in \{1, \ldots, 10\} \text{ mm},$$

$$p \in \{0.1, 0.2, \ldots, 2.0\}.$$  

The optimization was done as follows: first, the algorithm
finds the rotation time that yields the minimal achievable $w$
for a given heart rate. In the second step, it looks for a com-
bination of $S$ and $p$ to minimize the $z$-extent $(1 + pn_{\text{max}})S$
and thus to optimize the $z$-resolution. If there are more combina-
tions of slice thickness and pitch that result in the same
maximal $z$-resolution, then the combination with lowest pitch
is selected to improve (7d). Therefore we have introduced
the parameter $q \in [0, 1]$. It is defined by $p = qf_{Ht_{\text{rot}}} / (1 - w)$
and quantifies (7d). Low values for $q$ are desirable. The re-
The optimization was done as follows: first \( t_{\text{rot}} \) was selected to minimize \( w \), then the parameter \( l = (1 + p_{\text{lat}})S \), which describes the \( z \)-extent of the data contributing to a reconstruction and thus is related to the FWHM value, was minimized subject to the boundary conditions \( v = \pm 4 \text{ mm/s} \) and \( q \leq 0.9 \). The quality parameter \( q \), defined with \( p = q f_{\text{lat}}(1 - w) \), describes the slice condition \( p = f_{\text{lat}}(1 - w) \), Eq. (7d), quantitatively. In the lower part of the table, we additionally allowed a 0.5 s rotation time. Values for \( w \) and \( q \) rounded, if necessary.

<table>
<thead>
<tr>
<th>( f_{\text{H}} ) [min(^{-1})]</th>
<th>( S ) [mm]</th>
<th>( t_{\text{rot}} ) [s]</th>
<th>( p )</th>
<th>( w )</th>
<th>( q )</th>
<th>( l ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>4</td>
<td>0.75</td>
<td>0.8</td>
<td>0.38</td>
<td>0.80</td>
<td>10.4</td>
</tr>
<tr>
<td>60</td>
<td>3</td>
<td>0.75</td>
<td>1.0</td>
<td>0.36</td>
<td>0.86</td>
<td>9.0</td>
</tr>
<tr>
<td>70</td>
<td>3</td>
<td>1.25</td>
<td>1.7</td>
<td>0.46</td>
<td>0.63</td>
<td>13.2</td>
</tr>
<tr>
<td>80</td>
<td>3</td>
<td>1.00</td>
<td>1.4</td>
<td>0.33</td>
<td>0.70</td>
<td>11.4</td>
</tr>
<tr>
<td>90</td>
<td>2</td>
<td>1.00</td>
<td>2.0</td>
<td>0.50</td>
<td>0.67</td>
<td>10.0</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>0.75</td>
<td>1.5</td>
<td>0.43</td>
<td>0.68</td>
<td>8.0</td>
</tr>
<tr>
<td>110</td>
<td>2</td>
<td>0.75</td>
<td>1.5</td>
<td>0.38</td>
<td>0.68</td>
<td>8.0</td>
</tr>
<tr>
<td>120</td>
<td>2</td>
<td>0.75</td>
<td>1.5</td>
<td>0.50</td>
<td>0.50</td>
<td>8.0</td>
</tr>
</tbody>
</table>

The reader should be aware that these settings are determined to minimize the width \( w \) and thus the temporal resolution. Further on they are valid only for patients with constant heart rate only since small variations in \( f_{\text{H}} \) can result in discontinuous changes in the parameters. Moreover, it is not clear whether a minimal \( w \) yields the best images. Some of the reconstructions presented in Secs. III and IV were done with \( w \) above the minimal possible width. Hence, for this first study we chose to use the constant settings of \( S = 3 \text{ mm}, \ p = 1 \), and \( t_{\text{rot}} = 0.75 \text{ s} \) as mentioned at the beginning and decided to select the parameters \( o \) and \( w \) retrospectively to find good results.

### 4. Image noise of 180°CI

The cardio interpolation algorithm 180°CI is a modification of the standard \( z \)-interpolation algorithm 180°LI: Instead of requiring data from forbidden data ranges the next available data point falling into an allowed interval is used. The number of data points, and thus the number of independent random variables contributing to a certain interpolated data set and to the corresponding image, remains the same as in the 180°LI case. This means that pixel noise does neither increase nor decrease as compared to the standard linear interpolation (for a thorough discussion of image noise in standard spiral CT see Refs. 3, 6, and 7).

### 5. Algorithm 180°CD

The algorithm 180°CD (Cardio Delta) aims to reduce the effective scan time by doing a partial scan reconstruction. A data range of only \( 180^\circ + \delta \) (with \( \delta \leq \Phi \)) is effectively used and no \( z \)-interpolation is done.

The interpolated (planar) data at position \( z_R \) are given by

\[
P_{\text{CD}}(\beta, \alpha, z_R) = P(\beta_k, \alpha_k)
\]

with

\[
k = \arg \min_k |\alpha_k - \alpha_R|.
\]

\( k \) choses the projection which is closest to the reconstruction plane, i.e., it minimizes \( |\alpha - \alpha_R| \). This is depicted in Fig. 5.

The effective scan time depends on \( \delta \) which is determined by the object dimensions and the reconstruction position \( z_R \). Figure 6 shows the sinogram of a typical body scan within a range of \( 180^\circ + \Phi \). As can be seen, not all data necessary for doing the partial scan reconstruction lie within the support of the object function. In particular, data measured before reaching point \( A \) and after having reached point \( B \) will contribute only zeroes to the image aligned at \( \alpha_R \). Therefore with the corresponding detector positions

\[
\alpha_A = \inf \{ \alpha \mid P(\beta, \alpha) > 0, \ \alpha + \beta = \alpha_R - \pi/2 \},
\]

\[
\alpha_B = \sup \{ \alpha \mid P(\beta, \alpha) > 0, \ \alpha + \beta = \alpha_R + \pi/2 \},
\]

the angle \( \delta \) results as \( \delta = \alpha_B - \alpha_A - \pi \). We emphasize again that this value depends on \( \alpha_R \) (except when the object support is a cylinder centered on the axis of rotation) and on the object dimensions. In order to minimize variations of \( \delta \) as a function of \( \alpha_R \) the patient’s heart should be centered about the \( z \)-axis.

Assuming typical dimensions of a thorax cross-section of 25 cm \( \times \) 40 cm, for example, 50% \( \times \) 80% of the field of measurement (FOM) are used in our geometry; \( \delta \) varies between 26° and 42°, depending on the reconstruction position. Thereby effective scan times range from 0.57 \( \text{rot} \) to 0.62 \( \text{rot} \), i.e., 430 ms to 463 ms for a 0.75 s rotation.

Assuming that only the heart moves, the time of inconsistent data acquisition (i.e., the time while the heart contributes...
to the required data) is even lower: typical (transaxial) dimensions of the heart lie between 10 cm (20% of FOM or 10° when viewed from the source) and 15 cm (30% of FOM or 16°). An analogous consideration shows that only data within a range of 180° + 10° (180° + 16°, respectively) contain inconsistencies due to heart motion. This corresponds to an effective acquisition time between 395 ms and 408 ms in our case and this is exactly the effective temporal resolution that can be achieved with 180°CI. Of course, the algorithm requires more data for a complete set, but these data are not influenced by motion anymore and thus do not contribute to the effective acquisition time.

6. Prediction of 180°CD slice sensitivity profile

As compared to 180°CI, the SSP calculation is relatively simple and even the slice quality descriptors FWHM (full width at half-maximum), FWTM (full width at tenth maximum), FWTIA (full width at tenth area), and SPQI (slice profile quality index: area within FWHM limits/total SSP area)\(^2\) can be given explicitly. Due to the function \(\delta(\beta)\) in (2) the reconstruction breaks down to an averaging over the detector positions:

\[
\text{SSP}(\alpha_R) = \frac{\pi}{2} \int_{\frac{\alpha_R - \pi/2}{\alpha_R + \pi/2}} d\alpha \Pi \left( \frac{\alpha}{\Delta \alpha_S} \right) = \Pi \left( \frac{\alpha_R}{\pi} \right) \star \Pi \left( \frac{\alpha_R}{\Delta \alpha_S} \right)
\]

\[
= \Pi \left( \frac{\alpha_R}{\Delta \alpha_S} \right) \left( \alpha_R \right).
\]

We have used Eq. (8) to calculate the limits of integration. From the explicit formula for the convolution given in Appendix B, Eq. (B2), we find for the theoretical slice quality descriptors

\[
\text{FWHM} = \frac{1}{2} S(p \vee 2),
\]

\[
\text{FWTM} = \frac{1}{2} S(p + 2 - \frac{1}{2}(p \wedge 2)),
\]

\[
\text{FWTA} = \frac{1}{2} \left\{ \begin{array}{ll}
  S(p + 2 - \sqrt{2p}) & \text{if } (p \vee 2) < 10(p \wedge 2) \\
  \frac{1}{10}(p \vee 2) & \text{otherwise}
\end{array} \right.
\]

\[
\text{SPQI} = 1 - \frac{1}{3} \frac{p \wedge 2}{p \vee 2}.
\]

The SSP itself is a trapezium with a plateau width of \(S(\frac{1}{3}p - 1)\) and a base width of \(S(\frac{1}{3}p + 1)\). Obviously, the FWHM is equal to the slice thickness as long as the pitch is lower than 2 (for comparison: the full width at half-maximum of the standard linear interpolation 180°LI is equals \(S\) for pitch values smaller than 1, for \(1 \leq p \leq \frac{2}{3}\) it yields \(S(p + 1 - \sqrt{2p - 1})\) and we have \(S(\frac{1}{3}p + \frac{1}{2})\) for even higher pitch settings).

7. Image noise of 180°CD

Since the cardio delta algorithm 180°CD is a partial scan reconstruction (i.e., a next neighbor interpolation) the noise characteristics are the same as those of a conventional scan where only half of the data as compared to a full 360° scan are used without partial scan ray weighting. Thus image noise rises by a factor of \(\sqrt{2}\).

B. Patient examinations and data evaluation

Ten patients scheduled for a cardiac CT exam underwent spiral CT scanning for 30 s to 40 s with 0.75 s per 360° rotation. Simultaneous ECG recording was the only additional step added to the routine protocol. The raw data were stored and processed independent of the clinical routine. Three algorithms were applied for comparison: 180°LI (standard \(z\)-interpolation), 180°CI, and 180°CD. Images were reconstructed with an increment of 1 mm or less.

Data processing was as follows: Data manipulation, i.e., modified \(z\)-interpolation, was done on a standard PC. Reconstruction of the generated raw data and of the completed spiral, respectively (see Appendix A), was done on a PCTOMO station (MIR GmbH, Mührendorf, Germany) because of its high reconstruction speed (approximately 1 s per image) and its image post-processing capabilities. The two computers were connected via local network.

180°CI was, as mentioned above, implemented as a spiral completion algorithm (Appendix A) to reduce the amount of operations needed.

180°CD was implemented on the standard PC since PCTOMO has no option to do a partial scan reconstruction. Both algorithms take as an argument the ECG synchronization information which is used to provide information about the relative position within the current heart phase for each reconstruction position \(z_R\).
Our results are limited to the retrospective evaluation of ten randomly selected patient studies. The resulting images were appraised qualitatively only to receive a first impression. Nevertheless, the findings reported here were found consistent. Figure 7 shows five images for each algorithm in one patient study. They were reconstructed with 1 mm increments. The upper, middle, and lower row show 180°LI, 180°CI, and 180°CD reconstructed images, respectively. The first and the last columns apparently depict the heart in a phase of relatively slow motion: all algorithms produce comparable results regarding the anatomy and the relatively sharp delineation of the calcification. However, only the ECG-oriented algorithm 180°CI depicts the calcification clearly throughout the anatomic range covered during one complete heart cycle. Not only the first and the last image show the calcification and the anatomy correctly, but also the images inbetween do hardly suffer from motion.

**Fig. 7.** Comparison of the three algorithms (a). Upper row: 180°LI; middle row: 180°CI; lower row: 180°CD. Images are reconstructed at successive positions with an increment of 1 mm. Apparently, the 180°CI interpolation shows the anatomy best in its contiguity. E.g., observe how the calcification (arrows) grows continuously from slice to slice in 180°CI, which is not represented properly for the two other algorithms. The corresponding SSP plot for 180°CI is shown in (b). (Patient’s heart rate: 53 min⁻¹; o = 35%, w = 43%.)
artifacts. The steady change in the size of the calcified structures from slice to slice appears to represent the anatomy correctly in the 180°CI images, which is not the case either for 180°LI or for 180°CD.

The effective temporal resolution \( w/f_H \) for 180°CI that has been achieved lies between 400 ms and 500 ms and thus are comparable to the expected temporal resolution of 180°CD.

The artifact content was generally increased, however, in both ECG-correlated reconstructions, mostly by parallel streaks. The reasons are not fully clarified yet. For 180°CI these may be due to the fact that the algorithm switches from data segment to data segment depending on the ECG. We have not built in means to smooth the transition. The partial scan reconstruction, 180°CD (lower row), produces similar artifacts as in the standard case (180°LI), in particular at the heart borders, but also parallel streaks similar to those, but more dominating than in 180°CI. This is the typical behavior when reconstructing spiral data without \( z \)-interpolation.\(^7\)

While the transverse images showed more artifacts, multiplanar reformations were of improved quality: the 180°LI [Fig. 8(a)] algorithm necessarily produces motion artifacts at the edge of the heart which are not visible in the 180°CI case [Fig. 8(b)] as the cardiac interpolation images represent the same cardiac phase and thereby provides smooth continuous contours in multiplanar reformation (MPR) images.

### IV. DISCUSSION AND CONCLUSIONS

CT imaging of the heart has always been limited due to the fact that scan times are much longer than even the dia-
tole phase. Efforts at prospective ECG-gated data acquisition\(^8\) showed limited success and were not continued. Electron-beam tomography\(^9\) with scan times down to 50 ms appeared to be the only approach to reliable imaging of the heart by CT. The advent of spiral CT in combination with scanners capable of subsecond rotations provided the motivation to revisit this situation. This is of particular interest since, if successful, our approach can be made available on standard spiral scanners without major modifications.

The presented cardiac algorithms show promising results, with 180°CI in its present implementation the most favorable one. Although artifacts are still present, some structures are much more clearly delineated, as for example coronary calcifications. Figures 9 and 10 give a further impression of the superior quality in comparison to the standard \(z\)-interpolation 180°LI: calcifications that cannot be detected at all (Fig. 9) or are shown twice (Fig. 10) by the standard method are depicted sharply by 180°CI (arrows). This is important for the quantification of calcium which is usually done via a thresholding process\(^10\) and which has recently been tried for spiral scanning at 1 s rotation time without any modification of the spiral reconstruction algorithms.\(^11\) However, calcifications in images which suffer from motion will more likely fall below the threshold used (typically 130 HU\(^12\)) unless special efforts are undertaken. In addition, our new algorithm is inherently suited for volume representations, e.g., multiplanar reformations as demonstrated in Fig. 8 or 3D displays. Nevertheless, image quality of the cardiac interpolation algo-

![Fig. 9](image-url)
rithm strongly depends on the choice of the parameters $o$ and $w$. For width $w$ we have found several constraints (6) (see also Fig. 4) that determine a certain range within which the operator can navigate $w$. To find an optimal setting for the offset $o$, further work and clinical experience are necessary.

The partial scan reconstruction $180^\circ$CD does not lead to a continuous series of images in $z$-direction since it depicts, snapshotlike, the instantaneous position of the heart. Therefore successive images, selected by $z$-increments, do not necessarily contain data from corresponding heart phases. Alternatively we can prospectively select the reconstruction position based on the ECG information, i.e., by selecting the parameter $o$. This yields images for the same heart phase, but with varying $z$-increments. (A similar, but retrospective approach has been presented in Ref. 13.)

The artifact behavior of both approaches, $180^\circ$CI and $180^\circ$CD, is not optimal yet. The selection of data from different heart phases ($180^\circ$CI) and the nearest neighbor interpolation ($180^\circ$CD) both lead to discontinuities which result in parallel streaks in the images (see Fig. 7). We will try to overcome these problems by designing more smooth transitions from one heart phase to the other.

In conclusion, by introducing new algorithms we were able to demonstrate improvements in heart imaging due to ECG-correlated selection of segments ($180^\circ$CI) and reduced effective scan times ($180^\circ$CD), respectively. These new approaches may provide potential for new applications, e.g.,
for coronary calcium measurements. Future work is necessary to suppress artifacts introduced by the new algorithms. Clinical studies as well as the design of a cardiac phantom are necessary to further validate our approach. Significant improvements would become available by the development of future CT systems with higher and adjustable rotation speed to reduce motion effects and with multirow detector systems providing more adequate data for interpolation or nearest neighbor selection.

ACKNOWLEDGMENT

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APPENDIX A: SPIRAL COMPLETION IMPLEMENTATION OF 180°CI

Instead of implementing the algorithm as specified in Eq. (1), i.e., synthesizing planar raw data, there is the possibility of just completing the measured data which can then be processed in the normal fashion. Let \( \hat{P} \) denote corrected spiral data and let us assign for all \( \beta \) and \( \alpha \)

\[
\hat{P}(\beta, \alpha) = w' P(\beta_{k'}, \alpha_{k'}) + w'' P(\beta_{k''}, \alpha_{k''}),
\]

where

\[
k' = \sup\{k | \alpha_k < \alpha, \quad \alpha_k \in L_{o,w}\},
\]

\[
k'' = \inf\{k | \alpha_k \geq \alpha, \quad \alpha_k \in L_{o,w}\},
\]

\[
w' = 1 - w'' = \frac{\alpha_{k''} - \alpha}{\alpha_{k''} - \alpha_{k'}},
\]

Suppose \( \alpha \in L_{o,w} \). Then \( k'' = 0, \quad w'' = 1 \), and \( \hat{P}(\beta, \alpha) = P(\beta, \alpha) \forall \beta \), which means that data in allowed ranges are left unchanged. On the other hand, when \( \alpha \in L_{o,w} \), the corresponding projection undergoes a linear interpolation analogous to Eq. (1).

After completion, 180°LI z-interpolation is applied to \( \hat{P} \) to get planar data at \( z = z_R (\alpha_R, \text{ respectively}) \):

\[
P_{LI}(\beta, \alpha, \alpha_R) = w' P(\beta_{k'}, \alpha_{k'}) + w'' \hat{P}(\beta_{k''}, \alpha_{k''}),
\]

with

\[
k' = \sup\{k | \alpha_k < \alpha_R\},
\]

\[
k'' = \inf\{k | \alpha_k \geq \alpha_R\} = k' + 1,
\]

\[
w' = 1 - w'' = \frac{\alpha_{k''} - \alpha_R}{\alpha_{k''} - \alpha_{k'}}.
\]

Inserting (A1a) into (A2) gives

\[
P_{LI}(\beta, \alpha, \alpha_R) = w'(w_1' P(\beta_{k'_1}, \alpha_{k'_1}) + w'' P(\beta_{k''_1}, \alpha_{k''_1}))
\]

\[
+ w''(w_2' P(\beta_{k'_2}, \alpha_{k'_2}) + w'' P(\beta_{k''_2}, \alpha_{k''_2})),
\]

\[
(A3)
\]

Without loss of generality and for the sake of convenience, we have slightly changed the definition of \( k'_1 \) and \( k''_1 \): whenever case \( \alpha_k = \alpha_{k'} \), occurs, it is met by \( k'_1 \) and not by \( k''_1 \). Now we have \( k'_1 = k'_2 \) as well as \( k''_1 = k''_2 \). Equation (A3) gives

\[
P_{LI}(\beta, \alpha, \alpha_R)
\]

\[
= (w' w_1' + w'' w_2') P(\beta_{k'_1}, \alpha_{k'_1})
\]

\[
+ (w' w'' + w'' w_2') P(\beta_{k''_1}, \alpha_{k''_1})
\]

\[
= \frac{\alpha_{k''_1} - \alpha_R}{\alpha_{k''_1} - \alpha_{k'_1}} \frac{\alpha_{k''_1} - \alpha}{\alpha_{k''_1} - \alpha_{k'_1}} P(\beta_{k''_1}, \alpha_{k''_1}) + \left(1 - \frac{\alpha_{k''_1} - \alpha_R}{\alpha_{k''_1} - \alpha_{k'_1}} \right) P(\beta_{k''_1}, \alpha_{k''_1}),
\]

which is the same as (1). Thus we have proven that the spiral completion version is equivalent to the 180°CI algorithm.

APPENDIX B: EXPlicit Expressions

The calculation of slice sensitivity profiles requires to perform multiple convolutions of rectangle functions \( \Pi(\cdot) \) with different widths. For example, the 360°LI slice sensitivity
profile \( \text{SSP}(z) = \Lambda(z/d) \ast \Pi(z/S) \) is a convolution of a triangle function and a rectangle function as described in Ref. 3. The triangle function itself can be written as a self convolution of two rectangle functions.

Let \( 2a, 2b, \) and \( 2c \) denote the widths of three rectangle functions. To simplify the expressions we assume the widths to be positive and ordered: \( 2a \geq 2b \geq 2c > 0 \). A quite lengthy but noninstructive calculation leads to

\[
\begin{align*}
\Pi_{2a, 2b, 2c}^{**}(x) := & \Pi \left( \frac{x}{2a} \right) \ast \Pi \left( \frac{x}{2b} \right) \ast \Pi \left( \frac{x}{2c} \right) \\
= & \lim_{c \to 0} \frac{1}{2c} \Pi_{2a, 2b, 2c}^{**}(x) \\
= & \begin{cases} 
0 & \text{if } a + b + c < |x| \\
\frac{1}{2}(a + b + c - |x|)^2 & \text{if } a + b + c < |x| \leq a + b + c \\
2c(a + b - |x|) & \text{if } a - b + c < |x| \leq a + b - c \\
4bc - \frac{1}{2}(a - b - c - |x|)^2 & \text{if } |x| < (a - b - c) \\
4bc - (a - b - c)^2 - x^2 & \text{if } |x| \leq a - b - c.
\end{cases}
\end{align*}
\]  

(B1)

To obtain the convolution of two rectangle functions we use

\[
\lim_{c \to 0} \frac{1}{2c} \Pi \left( \frac{x}{2c} \right) = \delta(x),
\]

and get (for \( 2a > 2b > 0 \))

\[
\Pi_{2a, 2b}^{**}(x) := \Pi \left( \frac{x}{2a} \right) \ast \Pi \left( \frac{x}{2b} \right)
\]

\[
= \lim_{c \to 0} \frac{1}{2c} \Pi_{2a, 2b, 2c}^{**}(x) \\
= \begin{cases} 
0 & \text{if } a + b < |x| \\
a + b - |x| & \text{if } a - b < |x| \leq a + b. 
\end{cases}
\]

(B2)